A Novel Approach for Blind Estimation of a MIMO Channel Including Phase Unwrapping Ambiguity Elimination

Xia Liu
Kordia Solutions Pty Ltd
Banyo, Australia, QLD 4014
leo.liu@kordia.com.au

Hui Miao and Xiaodi Huang*
School of Computing and Mathematics
Chales Sturt University
Albury, Australia, NSW 2640
mzzhmh@gmail.com, xhuang@csu.edu.au

Abstract

In this paper, a novel method for blind estimation of a multiple input multiple output (MIMO) channel is described. Opposite to the conventional blind estimation algorithms that use singular value decomposition (SVD) or eigenvalue decomposition (EVD), it employs a simple coding scheme and a square root algorithm to recover the channel state information (CSI). The block coding scheme accompanying the proposed estimation approach requires a block encoder only at the transmitter. The proposed block coding scheme is spectral efficient as it offers the full coding rate when the numbers of transmitting and receiving antennas are equal. When only two transmitting and two receiving antennas are used, the coding scheme reduces the noise power to half of the original noise power. The phase ambiguity accompanying the blind approach to channel estimation is eliminated with only one symbol pilot sequence.

Keywords: MIMO system, Channel Estimation, Phase Ambiguity Elimination, Singular Value Decomposition, Eigenvalue Decomposition.

1 Introduction

Multiple Input Multiple Output (MIMO) signal transmission schemes are attractive for high-speed data transmission in wireless communication systems because they offer an increased data throughput (capacity) without increasing operational bandwidth [11], [3]. Also they can enhance the quality of signal transmission through the use of transmitter or receiver diversity. These advantages are possible under the condition that the MIMO channel state information (CSI) is available at the receiver. Traditionally CSI can be acquired by sending training sequences (pilot signals) as a header or within a block of transmitted symbols. An adverse outcome is that the pilot sequence takes up the precious bandwidth. In order to save the bandwidth and increase spectral efficiency, blind and semi-blind channel estimation methods can be used to obtain the CSI.

Several blind channel estimation methods have been described in [9] [12]. These methods are based on the subspace algorithm [8], which utilizes the orthogonality between the Sylvester matrix-formed channel matrix and noise subspace. There are several drawbacks of subspace-based MIMO channel estimation methods. Firstly, they suffer from so-called multi-dimensional ambiguity. Several training symbols are necessary to eliminate this ambiguity; Secondly, in order to compensate for extra degrees of freedom in the noise subspace when the number of transmit antennas are smaller than the number of antennas at the receiver, they require pre-coding [9] [12]. Thirdly, the operation of eigenvalue decomposition is required to recover the channel matrix. This leads to high implementation complexities.

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*Corresponding author: Web: http://csusap.csu.edu.au/~xhuang/, Tel: +61-02-60519652

In [4] [5], a semi-blind channel estimation approach employing orthogonal pilot maximum likelihood (OPML) estimator has been proposed. The approach performs SVD to the received signal correlation matrix estimating the 'whitening' matrix of the channel. Using the information of the 'whitening' matrix, the OPML estimator shows a 1dB improvement of bit error rate (BER) compared to the conventional least squares (LS) training scheme if the same length of training sequence is used. However, it still requires a considerable amount of training symbols to achieve the same performance as LS. Furthermore, SVD has to be applied twice to obtain the 'whitening' matrix and rotation matrix. This leads to the increased computational complexity.

The work in [7] presents a new singular value decomposition (SVD) blind channel estimation scheme employing a simple block pre-coding structure. The advantage of the presented approach is that CSI can be recovered without ambiguity if the proper modulation is applied. Another advantage of this scheme is that no block decoder is needed at the receiver. However, these advantages are accomplished at the expense of the coding rate. As more transmitting antennas are used the coding rate decreases. Therefore, the proposed scheme in [7] is suitable only for small size MIMO systems such as a 2x2 MIMO system to avoid a waste of precious bandwidth.

In this paper, we propose a novel blind channel estimation algorithm, which is of much lesser complexity than those based on SVD or EVD. Its superiority is that it conserves the advantages of the coding scheme described in [7] without sacrificing the coding rate. The new coding scheme offers a full coding rate (coding rate is equal to 1) for any number of transmitting antennas provided that it is equal to the number of receiving antennas. In the 2x2 MIMO system case, this coding scheme suppresses the noise power to half of the original noise power. In addition, the solution to eliminate the phase ambiguity using only one symbol pilot sequence is given. The rest of the paper is organized as follows. In section II, a model of MIMO system employing a block coding scheme is introduced. A new blind channel estimation method is described in section III. The solution to phase ambiguity elimination is given in section IV. Simulation results are presented in section V. Section VI concludes the paper.

2 System Description and Coding Scheme

In this paper, narrow band block fading channel is assumed. The number of transmitting and receiving antennas is denoted as N_i and N_r, respectively. Thus the channel H is described by the $N_i \times N_r$ matrix with h_{ij} entries representing the complex response between the i-th receive antenna and the j-th transmitting antenna. In further considerations N_t is assumed to be equal to Nr. During data transmission, the power is equally distributed to the transmitting antennas. The input symbols at transmitter are represented by

$$X = \{x_1, x_2, x_3, ...\} \tag{1}$$

where X stands for the independent identically distributed (i.i.d) Gaussian random signals with zero mean and variance matrix given by

$$E\{x_n {x_m}^H\} = \begin{cases} \sigma_s \ , & n = m \\ 0 \ , & n \neq m \end{cases}$$

where E{} implies the expectation and ${\sigma_s}^2$ is the power accompanying one symbol. The input symbols are encoded using a block coding encoder structure before being transmitted. As a result, the i-th symbol block is an element of matrix group $A_i \in C^{N_r \times N_r N_t}$. At the receiver, the collected data is affected by the properties of the channel as well as by an additive noise. Therefore the relationship between the transmitted encoded symbols and the received data is given as:

$$Y_i = HA_i + N_i \tag{2}$$

where Y_i is an $N_i \times N_r N_t$ received signal matrix and N_i is an $N_i \times N_r N_t$ (i.i.d) Gaussian random noise matrix with zero mean.

In the next step, a suitable coding scheme is introduced. In order to simplify considerations, the case of a 2x2 MIMO system is considered first. The extension to the case of $N_t \times N_t$ MIMO system is straightforward and shown next.

2.1 2X2 MIMO Case

The coded output of the transmitter can be written as:

$$A_i = \begin{bmatrix} A_{4i+1} & A_{4i+2} & A_{4i+3} & A_{4i+4} \end{bmatrix} \tag{3}$$

where

$$\begin{aligned}
A_{4i+1} &= diag(U_1)X_{2i+1}^T \\
A_{4i+2} &= diag(U_2)X_{2i+1}^T \\
\\
A_{4i+3} &= diag(U_1)X_{2i+2}^T \\
A_{4i+4} &= diag(U_2)X_{2i+2}^T \\
i &= 0,1,2,......,n
\end{aligned} \tag{4}$$

and $X_{2i+1} = \begin{bmatrix} x_{4i+1} & x_{4i+2} \end{bmatrix}$ and $X_{2i+2} = \begin{bmatrix} x_{4i+3} & x_{4i+4} \end{bmatrix}$. $U = \begin{bmatrix} U_1{}^T & U_2{}^T \end{bmatrix}$, $U_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $U_2 = \begin{bmatrix} 1 & -1 \end{bmatrix}$ represent the encoder structure. Therefore, the transmitted coded signals are:

$$A_{4i+1} = \begin{bmatrix} x_{4i+1} \\ x_{4i+2} \end{bmatrix}, A_{4i+2} = \begin{bmatrix} x_{4i+1} \\ -x_{4i+2} \end{bmatrix}$$

$$A_{4i+3} = \begin{bmatrix} x_{4i+3} \\ x_{4i+4} \end{bmatrix}, A_{4i+4} = \begin{bmatrix} x_{4i+3} \\ -x_{4i+4} \end{bmatrix} \tag{5}$$

or

$$A_i = \begin{bmatrix} x_{4i+1} & x_{4i+1} & x_{4i+3} & x_{4i+3} \\ x_{4i+2} & -x_{4i+2} & x_{4i+4} & -x_{4i+4} \end{bmatrix} \tag{6}$$

From expression (6), one can observe that 4 symbols are sent in 4 symbol periods during one block. Therefore, the code rate of the system is 1. The received signal blocks can be written as (7) for which an equivalent representation is given by (8):

$$Y_i = \begin{bmatrix} y_{11}^{2i+1} & y_{12}^{2i+1} & y_{11}^{2i+2} & y_{12}^{2i+2} \\ y_{21}^{2i+1} & y_{22}^{2i+1} & y_{21}^{2i+2} & y_{22}^{2i+2} \end{bmatrix}$$

$$= \begin{bmatrix} h_{11}x_{4i+1} + h_{12}x_{4i+2} & h_{11}x_{4i+1} - h_{12}x_{4i+2} & h_{11}x_{4i+3} + h_{12}x_{4i+4} & h_{11}x_{4i+3} - h_{12}x_{4i+4} \\ h_{21}x_{4i+1} + h_{22}x_{4i+2} & h_{21}x_{4i+1} - h_{22}x_{4i+2} & h_{21}x_{4i+3} + h_{22}x_{4i+4} & h_{21}x_{4i+3} - h_{22}x_{4i+4} \end{bmatrix} + N_i$$

$$= HA_i + N_i \tag{7}$$

in which $N_i = \begin{bmatrix} n_{11}^{2i+1} & n_{12}^{2i+1} & n_{11}^{2i+2} & n_{12}^{2i+2} \\ n_{21}^{2i+1} & n_{22}^{2i+1} & n_{21}^{2i+2} & n_{22}^{2i+2} \end{bmatrix}$ is the random Gaussian noise matrix.

$$\begin{matrix} y_{11}^{2i+1} = h_{11}x_{4i+1} + h_{12}x_{4i+2} + n_{11}^{2i+1} \\ y_{12}^{2i+1} = h_{11}x_{4i+1} - h_{12}x_{4i+2} + n_{12}^{2i+1} \end{matrix} \quad and \quad \begin{matrix} y_{21}^{2i+1} = h_{21}x_{4i+1} + h_{22}x_{4i+2} + n_{21}^{2i+1} \\ y_{22}^{2i+1} = h_{21}x_{4i+1} - h_{22}x_{4i+2} + n_{22}^{2i+1} \end{matrix}$$

$$\begin{array}{l} y_{11}^{2i+2} = h_{11}x_{4i+3} + h_{12}x_{4i+4} + n_{11}^{2i+2} \\ y_{12}^{2i+2} = h_{11}x_{4i+3} - h_{12}x_{4i+4} + n_{12}^{2i+2} \end{array} \; and \; \begin{array}{l} y_{21}^{2i+2} = h_{21}x_{4i+3} + h_{22}x_{4i+4} + n_{21}^{2i+2} \\ y_{22}^{2i+2} = h_{21}x_{4i+3} - h_{22}x_{4i+4} + n_{22}^{2i+2} \end{array} \tag{8}$$

By linking (8) directly to the individual channel matrix elements, one obtains:

$$h_{11}x_{4i+1} = \frac{y_{11}^{2i+1} + y_{12}^{2i+1}}{2} - \frac{n_{12}^{2i+1} + n_{11}^{2i+1}}{2}, h_{12}x_{4i+2} = -\frac{y_{12}^{2i+1} - y_{11}^{2i+1}}{2} + \frac{n_{12}^{2i+1} - n_{11}^{2i+1}}{2} \tag{9}$$

$$h_{21}x_{4i+1} = \frac{y_{21}^{2i+1} + y_{22}^{2i+1}}{2} - \frac{n_{22}^{2i+1} + n_{21}^{2i+1}}{2}, h_{22}x_{4i+2} = -\frac{y_{22}^{2i+1} - y_{21}^{2i+1}}{2} + \frac{n_{22}^{2i+1} + n_{21}^{2i+1}}{2} \tag{10}$$

$$h_{11}x_{4i+3} = \frac{y_{11}^{2i+2} + y_{12}^{2i+2}}{2} - \frac{n_{12}^{2i+2} + n_{11}^{2i+2}}{2}, h_{12}x_{4i+4} = -\frac{y_{12}^{2i+2} - y_{11}^{2i+2}}{2} + \frac{n_{12}^{2i+2} + n_{11}^{2i+2}}{2} \tag{11}$$

$$h_{21}x_{4i+3} = \frac{y_{21}^{2i+2} + y_{22}^{2i+2}}{2} - \frac{n_{22}^{2i+2} + n_{21}^{2i+2}}{2}, h_{22}x_{4i+4} = -\frac{y_{22}^{2i+2} - y_{21}^{2i+2}}{2} + \frac{n_{22}^{2i+2} - n_{21}^{2i+2}}{2} \tag{12}$$

a result, the relationship between the received and transmitted data is given by (13):

$$\overline{Y_i} = \overline{HX_i} + \overline{N_i} \tag{13}$$

in which the individual terms are identified by (14):

$$\begin{bmatrix} \frac{y_{11}^{2i+1}+y_{12}^{2i+1}}{2} \\ \frac{y_{11}^{2i+1}-y_{12}^{2i+1}}{2} \\ \frac{y_{11}^{2i+2}+y_{22}^{2i+2}}{2} \\ \frac{y_{21}^{2i+2}-y_{22}^{2i+2}}{2} \end{bmatrix} = \begin{bmatrix} h_{11} & 0 & 0 & 0 \\ 0 & h_{12} & 0 & 0 \\ 0 & 0 & h_{21} & 0 \\ 0 & 0 & 0 & h_{22} \end{bmatrix} \begin{bmatrix} x_{4i+1} \\ x_{4i+2} \\ x_{4i+3} \\ x_{4i+4} \end{bmatrix} + \begin{bmatrix} n_{4i+1} \\ n_{4i+2} \\ n_{4i+3} \\ n_{4i+4} \end{bmatrix} \tag{14}$$

where

$$\begin{array}{l} n_{4i+1} = \frac{n_{12}^{2i+1}+n_{11}^{2i+1}}{2}, n_{4i+2} = \frac{n_{11}^{2i+1}-n_{12}^{2i+1}}{2}, \\ n_{4i+3} = \frac{n_{22}^{2i+2}+n_{21}^{2i+2}}{2}, n_{4i+4} = \frac{n_{21}^{2i+2}-n_{22}^{2i+2}}{2} \end{array} \tag{15}$$

Due to the fact that the elements in N_i representing the random noise are all Gaussian, also the elements in $\overline{N_i}$ obey the Gaussian distribution. However, the average power of each element in $\overline{N_i}$ in equation (15) is half of that in N_i. This means that the coding scheme suppresses the noise power to half of its original value.

2.2 $N_r \times N_t$ MIMO Case

Now, the considerations are extended to the case of $N_r \times N_t$ MIMO system. As a result, the earlier introduced expression (3) converts to the new expression (16)

$$A_i = \begin{bmatrix} A_{N_rN_ti+1} & A_{N_rN_ti+2} & \dots & A_{N_rN_ti+N_rN_t} \end{bmatrix} \tag{16}$$

and U becomes

$$U = \begin{bmatrix} U_1{}^T & U_2{}^T & \dots & U_{Nt}{}^T \end{bmatrix} \tag{17}$$

Next, the equivalent of (4) becomes expression (18) as given by:

$$\begin{gathered}
A_{N_rN_ti+1} = diag(U_1)X^T_{N_ti+1} \\
A_{N_rN_ti+2} = diag(U_2)X^T_{N_ti+1} \\
\vdots \\
A_{N_rN_ti+N_t} = diag(U_{N_t})X^T_{N_ti+1} \\
A_{N_rN_ti+N_t+1} = diag(U_1)X^T_{N_ti+2} \\
A_{N_rN_ti+N_t+2} = diag(U_2)X^T_{N_ti+2} \\
\vdots \\
A_{N_rN_ti+2N_t} = diag(U_{N_t})X^T_{N_ti+2} \\
A_{N_rN_ti+2N_t+1} = diag(U_{N_t})X^T_{N_ti+3} \\
\vdots \\
A_{N_rN_ti+(N_r-1)N_t+1} = diag(U_1)X^T_{N_ti+N_r} \\
\vdots \\
A_{N_rN_ti+N_rN_t} = diag(U_{N_t})X^T_{N_ti+N_r}
\end{gathered} \tag{18}$$

where

$$\begin{gathered}
X_{N_ti+1} = [\; x_{N_rN_ti+1} \;\; x_{N_rN_ti+2} \quad \ldots \;\; x_{N_rN_ti+N_t} \;] \\
X_{N_ti+2} = [\; x_{N_rN_ti+N_t+1} \;\; x_{N_rN_ti+N_t+2} \quad \ldots \;\; x_{N_rN_ti+2N_t} \;] \\
X_{N_ti+3} = [\; x_{N_rN_ti+2N_t+1} \;\; x_{N_rN_ti+2N_t+2} \quad \ldots \;\; x_{N_rN_ti+3N_t} \;] \\
\cdots\cdots \\
X_{N_ti+N_r} = [\; x_{N_rN_ti+(N_r-1)N_t+1} \;\; x_{N_rN_ti+(N_r-1)N_t+2} \quad \ldots \;\; x_{N_rN_ti+N_rN_t} \;]
\end{gathered} \tag{19}$$

It becomes apparent from inspection of expressions (16)-(19) that converting the case of 2x2 MIMO system to the general case of $N_r \times N_t$ MIMO system is straight forward. In order to satisfy the proposed coding objective, the encoder structure $U = [\; {U_1}^T \quad {U_2}^T \quad \ldots \quad {U_{Nt}}^T \;]$ needs to be designed in this way that there is a sufficient amount of information to determine the relationship between $\overline{H}_i$ and $\overline{X}_i$. One possible solution to this problem is to use U in the form given by (20):

$$U = \begin{bmatrix} 1 & 1 & -1 & \cdots & -1 \\ 1 & -1 & 1 & \cdots & -1 \\ 1 & -1 & -1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 1 & -1 & \cdots & -1 & -1 \end{bmatrix}_{N_r \times N_t} \tag{20}$$

It becomes apparent that for the 2x2 MIMO case, U in (20) reduces to $U = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ or $U = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. The encoder structure design rule is such that the position of 1 in each row except the first row doesn't necessarily have to follow the diagonal pattern. However, the 1's position in each row must be different from the others'. Following this property, we can get similar relationships to those given by

(9-10) and (11-12) for elements in the first (N_t-1)-th columns from left to right of H. The N_r elements in terms of the N_t-th column are given as:

$$h_{1N_t}x_{N_rN_ti+1} = \frac{y_{11}^{N_ti+1}+y_{12}^{N_ti+1}\cdots+y_{1(N_t-2)}^{N_ti+1}+y_{1(N_t-1)}^{N_ti+1}+y_{1N_t}^{N_ti+1}}{2} - \frac{n_{11}^{N_ti+1}+n_{12}^{N_ti+1}\cdots+n_{1(N_t-2)}^{N_ti+1}+n_{1(N_t-1)}^{N_ti+1}+n_{1N_t}^{N_ti+1}}{2} \tag{21}$$

$$h_{N_rN_t}x_{N_rN_ti+N_r} = \frac{y_{N_r1}^{N_ti+N_r}+y_{N_r2}^{N_ti+N_r}\cdots+y_{N_r(N_t-2)}^{N_ti+N_r}+y_{N_r(N_t-1)}^{N_ti+N_r}+y_{N_rN_t}^{N_ti+N_r}}{2} - \frac{n_{N_r1}^{N_ti+N_r}+n_{N_r2}^{N_ti+N_r}\cdots+n_{N_r(N_t-2)}^{N_ti+N_r}+n_{N_r(N_t-1)}^{N_ti+N_r}+n_{N_rN_t}^{N_ti+N_r}}{2} \tag{22}$$

The relationship in equation (13) can still be applied here. But in the new case, $\overline{N_i}$ becomes the N_rN_tX1 vector. The power of the first $N_r(N_t-1)$ elements from top to bottom in $\overline{N_i}$ is half of the original noise. The power of each element of last N_r elements is $\frac{N_t-2}{2}$ of the original noise. The power of noise impacts the accuracy of channel estimation accuracy. Therefore, when the number of antenna elements at transmitter and receiver exceeds 2, the accuracy of estimation is significantly affected by the number of transmitting antennas.

3 Blind Channel Estimation

Blind channel estimation requires the knowledge of the correlation matrix of $\overline{Y}$, which is given by:

$$R_i = E\{\overline{Y_i}\overline{Y}_i^H\} = \overline{H}E\{\overline{X_i}\overline{X_i}^H\}\overline{H}^H + E\{\overline{N_i}\overline{N_i}^H\} \tag{23}$$

As concluded in the last section, the power of each element in $\overline{N_i}$ is half of that in the actual noise matrix N_i. Therefore equation (23) can be rewritten as:

$$R_i = E\{\overline{Y_i}\overline{Y}_i^H\} = \overline{H}E\{\overline{X_i}\overline{X_i}^H\}\overline{H}^H + E\{\overline{N_i}\overline{N_i}^H\} \tag{24}$$

If the information symbol sequence is of unit power then (24) becomes:

$$R_i = \overline{H}\overline{\overline{X_i}}\overline{H}^H + E\{\overline{N_i}\overline{N_i}^H\} \tag{25}$$

where

$$\overline{\overline{X}} = E\{\overline{X_i}\overline{X_i}^H\} = I_{N_rN_t\times N_rN_t} \tag{26}$$

As a result, (25) can be converted to (27)

$$R_i = \overline{H}\overline{H}^H + E\{\overline{N_i}\overline{N_i}^H\} \tag{27}$$

By introducing $\overline{h} = vec(H)$, the following holds:

$$\overline{H}\overline{H}^H = diag(\left|\overline{h}\right|^2) \tag{28}$$

where vec(.) means vectorize and $\|.\|$ denotes absolute value.

$$\left|\overline{h}\right|^2 = \begin{bmatrix} |h_{11}|^2 & |h_{12}|^2 & \cdots & |h_{N_rN_t}|^2 \end{bmatrix} \tag{29}$$

Then

$$R_i = diag(\left|\overline{h}\right|^2) + E\{\overline{N_i}\overline{N_i}^H\} \tag{30}$$

The solution to estimating the $\overline{h}$ is equivalent to finding the roots of the diagonal elements in R. To obtain the roots, the square-root algorithm can be applied. In the present case, the square roots represent the norms of the elements in channel matrix H. It is apparent that the square-root operation introduces the phase ambiguity to the estimated $\overline{h}$. In the next section, a method for eliminating the phase ambiguity is proposed.

4 Phase Ambiguity Elimination

In the proposed approach, the phase ambiguity is eliminated using only one symbol pilot sequence. In this case, the information is used both from the pilot sequence and the presented coding scheme. Similar to the approach presented in section II, this is demonstrated first for the case of a 2x2 MIMO system. The extension to the case of $N_t \times N_t$ MIMO system is straight forward and is shown next.

4.1 2x2 MIMO Case

For a 2x2 MIMO system, the information obtained from the blind estimation is the estimated norm of each element in channel matrix H.

$$\left|\hat{H}\right| = \begin{bmatrix} \left|\hat{h}_{11}\right| & \left|\hat{h}_{12}\right| \\ \left|\hat{h}_{21}\right| & \left|\hat{h}_{22}\right| \end{bmatrix} \tag{31}$$

By sending a pilot sequence P where

$$P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{32}$$

one obtains

$$Y_p = HP + N \tag{33}$$

in which Y_p and P are known. More specifically,

$$Y_p = \begin{bmatrix} X_p^1 + j \cdot Y_p^1 \\ X_p^2 + j \cdot Y_p^2 \end{bmatrix} = \begin{bmatrix} |h_{11}|\, e^{j\theta_{11}} & |h_{12}|\, e^{j\theta_{12}} \\ |h_{21}|\, e^{j\theta_{21}} & |h_{22}|\, e^{j\theta_{22}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \tag{34}$$

where

$$\begin{aligned} N_1 &= n_1^x + j * n_1^y \\ N_2 &= n_2^x + j * n_2^y \end{aligned} \tag{35}$$

By replacing $|h_{N_r N_t}|$ with the estimated ones from equation (31), the following is obtained

$$X_p^1 = \left|\hat{h}_{11}\right| \cos\theta_{11} + \left|\hat{h}_{12}\right| \cos\theta_{12} + n_1^x \tag{36}$$

$$Y_p^1 = \left|\hat{h}_{11}\right| \sin\theta_{11} + \left|\hat{h}_{12}\right| \sin\theta_{12} + n_1^y \tag{37}$$

$$X_p^2 = \left|\hat{h}_{21}\right| \cos\theta_{21} + \left|\hat{h}_{22}\right| \cos\theta_{22} + n_2^x \tag{38}$$

$$Y_p^2 = \left|\hat{h}_{21}\right| \sin\theta_{21} + \left|\hat{h}_{22}\right| \sin\theta_{22} + n_2^y \tag{39}$$

To determine θ_{11}, θ_{12}, θ_{21} and θ_{22} under the presence of noise requires equations (9), (10) and (11), (12) to be divided by sides ((9)/(10) and (11)/(12)) so that the following is obtained:

$$\begin{aligned} \frac{h_{11}}{h_{21}} &= \frac{|h_{11}|}{|h_{21}|} e^{j(\theta_{11}-\theta_{21})} = \frac{|h_{11}|}{|h_{21}|} e^{j\sigma_1} = \frac{\frac{y_{11}^{2i+1}+y_{12}^{2i+1}}{2} - \frac{n_{12}^{2i+1}+n_{11}^{2i+1}}{2}}{\frac{y_{21}^{2i+1}+y_{22}^{2i+1}}{2} - \frac{n_{22}^{2i+1}+n_{21}^{2i+1}}{2}}, \\ \frac{h_{12}}{h_{22}} &= \frac{|h_{12}|}{|h_{22}|} e^{j(\theta_{12}-\theta_{22})} = \frac{|h_{12}|}{|h_{22}|} e^{j\sigma_2} = \frac{\frac{y_{12}^{2i+1}-y_{11}^{2i+1}}{2} - \frac{n_{12}^{2i+1}+n_{11}^{2i+1}}{2}}{\frac{y_{22}^{2i+1}-y_{21}^{2i+1}}{2} - \frac{n_{22}^{2i+1}+n_{21}^{2i+1}}{2}} \end{aligned} \tag{40}$$

$$\begin{aligned} \frac{h_{11}}{h_{21}} &= \frac{|h_{11}|}{|h_{21}|} e^{j(\theta_{11}-\theta_{21})} = \frac{|h_{11}|}{|h_{21}|} e^{j\sigma_1} = \frac{\frac{y_{11}^{2i+2}+y_{12}^{2i+2}}{2} - \frac{n_{12}^{2i+2}+n_{11}^{2i+2}}{2}}{\frac{y_{21}^{2i+2}+y_{22}^{2i+2}}{2} - \frac{n_{22}^{2i+2}+n_{21}^{2i+2}}{2}}, \\ \frac{h_{12}}{h_{22}} &= \frac{|h_{12}|}{|h_{22}|} e^{j(\theta_{12}-\theta_{22})} = \frac{|h_{12}|}{|h_{22}|} e^{j\sigma_2} = \frac{\frac{y_{12}^{2i+2}-y_{11}^{2i+2}}{2} - \frac{n_{12}^{2i+2}+n_{11}^{2i+2}}{2}}{\frac{y_{22}^{2i+2}-y_{21}^{2i+2}}{2} - \frac{n_{22}^{2i+2}+n_{21}^{2i+2}}{2}} \end{aligned} \tag{41}$$

where $\sigma_1 = \theta_{11} - \theta_{21}$ and $\sigma_1 = \theta_{12} - \theta_{22}$. With equation (36) and (37), σ_1 and σ_2 can be estimated in the presence of noise. Then θ_{11}, θ_{21} and θ_{12}, θ_{22} can be expressed by each other with estimated σ_1 and σ_2 as,

$$\theta_{21} = \hat{\sigma}_1 - \theta_{11} \tag{42}$$

$$\theta_{22} = \hat{\sigma}_2 - \theta_{12} \tag{43}$$

By substituting (42) into equations (36), (37) and (43) into equations (38), (39), phases θ_{11}, θ_{21}, θ_{12} and θ_{22} can be determined in the presence of noise. The above derived expressions show that the phase ambiguity is resolved.

4.2 N_rXN_t MIMO Case

In order to obtain an extension to the case of a N_rXN_t MIMO system, the estimated norm of each element in channel matrix H is expressed as

$$|\hat{H}| = \begin{bmatrix} |\hat{h}_{11}| & \cdots & |\hat{h}_{1N_t}| \\ \vdots & \ddots & \vdots \\ |\hat{h}_{N_r1}| & \cdots & |\hat{h}_{N_rN_t}| \end{bmatrix} \tag{44}$$

The pilot sequence P then becomes

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{N_t} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{45}$$

In this case equation (45) is equivalent of (34),

$$Y_p = \begin{bmatrix} X_p^1 + j \cdot Y_p^1 \\ X_p^2 + j \cdot Y_p^2 \\ \vdots \\ X_p^{N_r} + j \cdot Y_p^{N_r} \end{bmatrix} = \begin{bmatrix} |h_{11}| e^{j\theta_{11}} & \cdots & |h_{1N_t}| e^{j\theta_{1N_t}} \\ \vdots & \ddots & \vdots \\ |h_{N_r1}| e^{j\theta_{21}} & \cdots & |h_{N_rN_t}| e^{j\theta_{N_rN_t}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_{N_r} \end{bmatrix} \tag{46}$$

where

$$\begin{aligned} N_1 &= n_1^x + j * n_1^y \\ N_2 &= n_2^x + j * n_2^y \\ &\vdots \\ N_{N_r} &= n_{N_r}^x + j * n_{N_r}^y \end{aligned} \tag{47}$$

By replacing $|h_{N_rN_t}|$ with the estimation values from equation (46),

$$X_p^1 = |\hat{h}_{11}| \cos\theta_{11} + |\hat{h}_{12}| \cos\theta_{12} \cdots + |\hat{h}_{1N_t}| \cos\theta_{1N_t} + n_1^x \tag{48}$$

$$Y_p^1 = \left|\hat{h}_{11}\right| \sin\theta_{11} + \left|\hat{h}_{12}\right| \sin\theta_{12} \cdots + \left|\hat{h}_{1N_t}\right| \sin\theta_{1N_t} + n_1^y \tag{49}$$

$$X_p^{N_r} = \left|\hat{h}_{N_r1}\right| \cos\theta_{N_r1} + \left|\hat{h}_{N_r2}\right| \cos\theta_{N_r2} \cdots + \left|\hat{h}_{N_rN_t}\right| \cos\theta_{N_rN_t} + n_{N_r}^x \tag{50}$$

$$Y_p^{N_r} = \left|\hat{h}_{N_r1}\right| \sin\theta_{N_r1} + \left|\hat{h}_{N_r2}\right| \sin\theta_{N_r2} \cdots + \left|\hat{h}_{N_rN_t}\right| \sin\theta_{N_rN_t} + n_{N_r}^y \tag{51}$$

To determine the phases θ requires similar operation as used for the 2x2 MIMO system. Therefore,

$$\begin{aligned}
\frac{h_{11}}{h_{21}} &= \frac{|h_{11}|}{|h_{21}|} e^{j(\theta_{11}-\theta_{21})} = \frac{|h_{11}|}{|h_{21}|} e^{j\sigma_{121}}, \\
\frac{h_{11}}{h_{31}} &= \frac{|h_{11}|}{|h_{31}|} e^{j(\theta_{11}-\theta_{31})} = \frac{|h_{11}|}{|h_{31}|} e^{j\sigma_{131}} \\
&\vdots \\
\frac{h_{11}}{h_{N_r1}} &= \frac{|h_{11}|}{|h_{N_r1}|} e^{j(\theta_{11}-\theta_{N_r1})} = \frac{|h_{11}|}{|h_{N_r1}|} e^{j\sigma_{1N_r1}} \\
&\vdots \\
\frac{h_{1N_t}}{h_{2N_t}} &= \frac{\left|h_{1N_t}\right|}{\left|h_{2N_t}\right|} e^{j(\theta_{1N_t}-\theta_{2N_t})} = \frac{\left|h_{1N_t}\right|}{\left|h_{2N_t}\right|} e^{j\sigma_{12N_t}} \\
\frac{h_{1N_t}}{h_{3N_t}} &= \frac{\left|h_{1N_t}\right|}{\left|h_{3N_t}\right|} e^{j(\theta_{1N_t}-\theta_{3N_t})} = \frac{\left|h_{1N_t}\right|}{\left|h_{3N_t}\right|} e^{j\sigma_{13N_t}} \\
&\vdots \\
\frac{h_{1N_t}}{h_{N_rN_t}} &= \frac{\left|h_{1N_t}\right|}{\left|h_{N_rN_t}\right|} e^{j(\theta_{1N_t}-\theta_{N_rN_t})} = \frac{\left|h_{1N_t}\right|}{\left|h_{N_rN_t}\right|} e^{j\sigma_{1N_rN_t}}
\end{aligned} \tag{52}$$

Then with estimated σ we have,

$$\begin{aligned}
\theta_{21} &= \hat{\sigma}_{121} - \theta_{11} \\
\theta_{31} &= \hat{\sigma}_{131} - \theta_{11} \\
&\vdots \\
\theta_{N_r1} &= \hat{\sigma}_{1N_r1} - \theta_{11} \\
&\vdots \\
\theta_{2N_t} &= \hat{\sigma}_{12N_t} - \theta_{1N_t} \\
\theta_{3N_t} &= \hat{\sigma}_{13N_t} - \theta_{1N_t} \\
&\vdots \\
\theta_{N_tN_t} &= \hat{\sigma}_{1N_rN_t} - \theta_{1N_t}
\end{aligned} \tag{53}$$

By applying (53) into equations (50) and (51), phases θ can be determined. Hence, the phase ambiguity is resolved. Following estimation of the channel matrix, the transmitted symbols X can be estimated using the following operation:

$$\hat{\tilde{X}}_i = \left[diag\left(vec(\hat{H})\right)\right]^{-1} \bar{Y}_i \tag{54}$$

where vec() denotes operation vectorize. It becomes apparent that when the channel estimation is accomplished, other coding schemes, like space time block coding (STBC) or space frequency block coding (SFBC), can be applied to the rest of transmitted signal symbols. These new schemes can be used to increase diversity gain.

5 Simulation Results

In order to validate the proposed MIMO channel estimation algorithm computer simulations are undertaken. To assess the performance of the proposed channel estimation method, the least square (LS) training-based channel estimation method is chosen as a reference. In this assessment, the mean square error (MSE) is used as given by

$$MSE = E\{\|H - \hat{H}\|_F^2\} \tag{55}$$

in which $\|.\|_F^2$ stands for the Frobenius norm. In the LS method, the estimated channel matrix can be written as [10] [6],

$$\hat{H}_{LS} = YP^{\dagger} \tag{56}$$

where $\{.\}\dagger$ stands for the pseudo-inverse operation. The MSE of LS method is obtained from

$$MSE_{LS} = E\{\|H - \hat{H}_{LS}\|_F^2\} \tag{57}$$

According to [1] and [2], the minimum value of MSE for the LS method is given by

$$MSE_{\min}^{LS} = \frac{M_t^2 M_r}{\rho} \tag{58}$$

in which ρ stands for the transmitted power to noise ratio in the training mode. Here we assume that the SNR in the proposed estimation scenario is given as ρ. Figure 1 shows the performance of the LS method and the proposed blind estimation method for a 2x2 MIMO system.

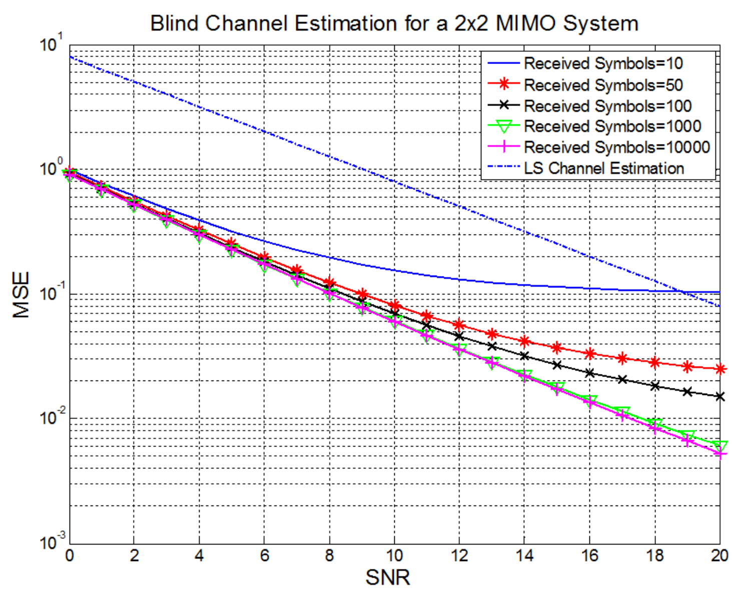

Figure 1: Channel performance of LS and proposed blind estimation method for a 2x2 MIMO system

One can see from Figure 1 that when the number of received symbols increases, the blind estimation accuracy is improved. Also seen in Figure 1 is that the performance of the blind channel estimation is always better than that offered by LS. This is mainly because the proposed algorithm reduces the noise power to half of the original. Figure 2 shows that the simulation results for the LS method and the proposed blind estimation method for a 4x4 MIMO system. The trend is similar to that observed in Figure 1 when LS and the new estimation method results are compared. In addition, one can see that as the number of transmitting and receiving antennas increases, the performance of LS method gets worse. However, the performance of the proposed method is almost unchanged.

Figure 3 and 4 demonstrate the convergence of the blind channel estimation for 2x2 and 4x4 MIMO channels for three different values of SNR. From the presented results it can be seen that the convergence

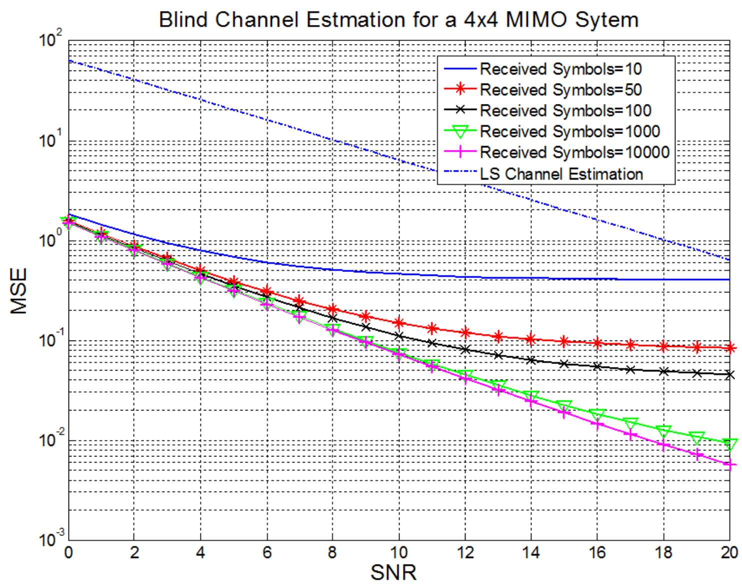

Figure 2: Channel performance of LS and proposed blind estimation method for a 4x4 MIMO system

rate improves sharply when the number of symbols increases from 1 to 100. For 100 to 500 symbols, the convergence improves slowly and then there is no further improvement when about 700 symbols is received. One can see that with 100 received symbols the estimation is already very accurate.

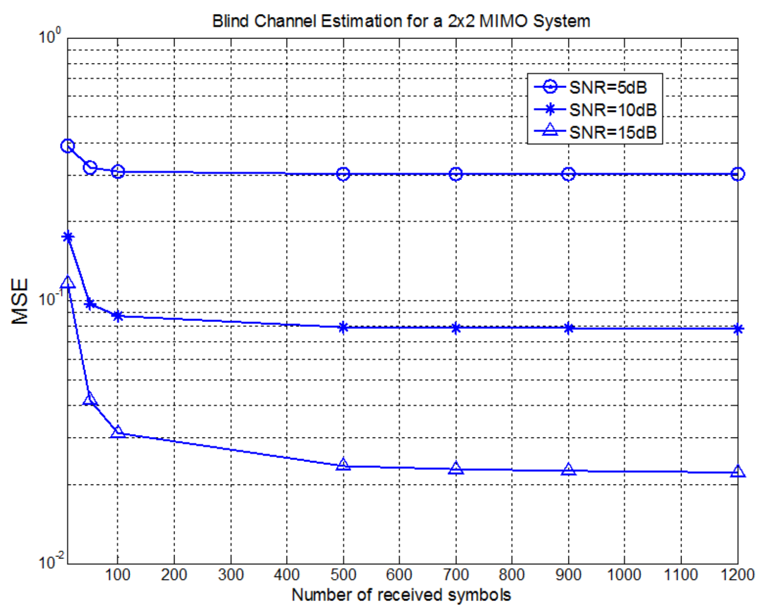

Figure 3: MSE versus number of received symbols at three different SNR values for a 2x2 MIMO system

The next investigations concern the influence of number of transmitting and receiving antenna element on channel estimation. Figure 5 presents the channel estimation results in terms of MSE of the proposed blind channel estimation and LS method versus the number of transmitting and receiving antennas. The earlier assumption of equal number of transmitting and receiving antennas is used.

One can see that as the number of transmitting/receiving antenna elements increases, the MSE of

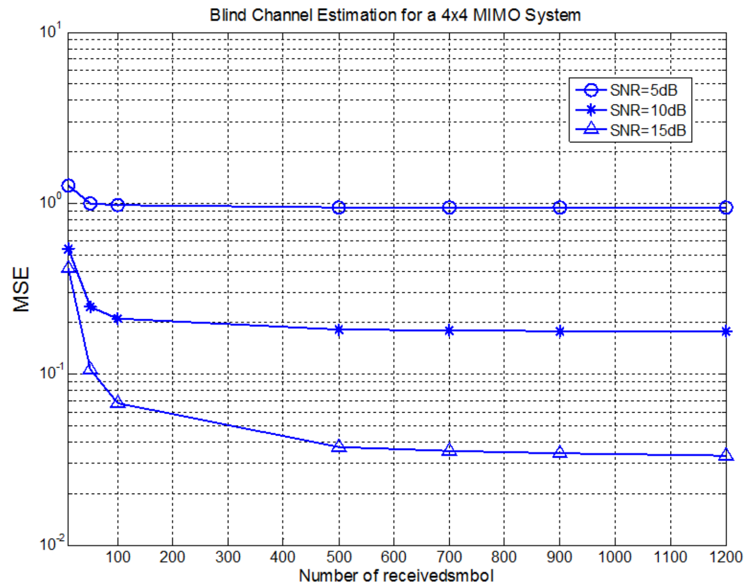

Figure 4: MSE versus number of received symbols at three different SNR values for a 4x4 MIMO system

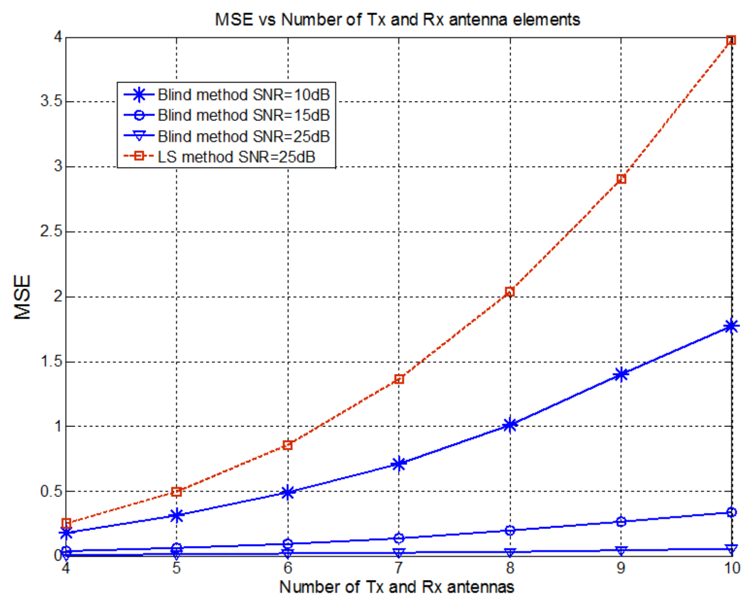

Figure 5: MSE versus number of transmit and receive antenna elements at different SNR values

the blind channel estimation method is increased. This phenomenon has already been reported for the training-based channel estimation methods. The finding is confirmed in Figure 5 for the LS method assuming SNR of 25dB. By comparing the LS and blind estimation method results, it becomes apparent that the blind estimation errors obtained for SNR of 10dB are still much lower than for the corresponding results when the LS training-based method operating at SNR of 25dB is used. The presented results clearly indicate that the proposed blind MIMO channel estimation method is less adversely affected by an increase of the number of transmitting and receiving antennas than the LS method.

6 Conclusion

In this paper a novel algorithm for blind MIMO channel estimation in conjunction with a suitable coding scheme and phase ambiguity elimination has been presented. The coding scheme exhibits high spectral efficiency when the number of transmitting is equal to the number of receiving antennas. Using the proposed coding scheme, the noise power for a 2x2 MIMO channel is reduced to the half of its original value. The proposed algorithm involves the square-root technique and shows low-level of processing complexity. The simulation results have shown a fast convergence rate for estimating the channel. The performance of this channel estimation algorithm is less adversely affected than the training-based schemes when the number of antennas is increased at the transmitter and receiver sides. These advantages have been proved for a slowly fading MIMO channel. Further advantages of the proposed blind channel estimation are expected for a fast fading channel, for which it is known that non-blind channel estimation methods would require further increases in pilot symbols.

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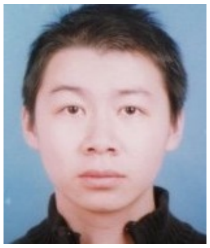
Author Biography



Xia Liu is currently a Project Engineer in Kordia. He received the Phd degree in Telecommunication in University of Queensland, Australia in 2011. And he received the Master of Engineering degree in University of Queensland in 2007. He received his B.E. degree in Zhejiang University of Technology, China 2005. His research interests are: MIMO systems, wireless ad-hoc networks, wireless sensor networks.

Hui Miao is a PhD student in the School of Computing and Mathematics at Charles Sturt University, Australia. He received the B.E. degree in Electronics Engineering in Southwest Jiaotong University, China in 2004. And he received the Master of Engineering degree in School of Information Technology and Electrical Engineering from The University of Queensland, Australia, in 2007. His major interests are mobile agents, network management, heuristic algorithms, smart grid and mobile networks.

Xiaodi Huang is a Senior Lecturer in the School of Computing and Mathematics at Charles Sturt University, Australia He received his PhD degree in 2004. His research areas include information visualization, networking and data mining. He has published over 50 scholar papers in international journals and conferences. Dr. Huang is a regular reviewer for server international journals, and serves as the committee members of international conferences. He is a member of the ACM and IEEE Computer Society. For details, please visit his homepage at `http://csusap.csu.edu.au/~xhuang`