An Investigation of Telecom Mobile Data Billing Plans

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Abstract

In the recent years, mobile operators have provided many billing alternatives such as *limited* and *unlimited* billing plans, and *shared* and *non-shared* data plans for the users with different needs. A non-shared data plan is designed for a single user with a limited monthly data allowance. On the other hand, the monthly data allowance of a shared data plan is shared by a group of users with multiple devices. The mobile operators often conduct the primary price study to compare their billing plans, which shows the relationship between the prices of the billing plans against the fixed amounts of data usage. Although the primary price study can easily and quickly draw the conclusions, it only provides rough billing plan suggestions. In reality, the amounts of data usage are not fixed, and therefore should be measured from commercial mobile networks to reflect the user behaviors on data usage. This paper proposes an analytical approach by using the measured data of Chunghwa Telecom Co., Ltd. (CHT), the largest telecommunications company in Taiwan, to derive the expected payments of various billing plans. The results of the analytical model are more accurate than those of the primary price study, and therefore provide better suggestions for billing plan selection. Other mobile operators can easily use our model to analyze the billing alternatives with their measured data.

Keywords: 3G, 4G LTE, mobile data service, unlimited and limited data billing, shared and non-shared data billing

1 Introduction

Access the Internet anywhere through a smartphone has become popular worldwide. Based on different data service requirements, a mobile operator may provide limited or unlimited billing plans, or billing plans for non-shared or shared data usage to satisfy users' needs. Unlimited billing plans are exercised for 3G service, which allow users to enjoy unlimited data usage [19]. Many mobile networks have evolved to 4G LTE, and the operators may have exercised limited data plans with monthly data allowances. This paper will focus on billing plans with limited data usage. In the limited billing domain, a non-shared data plan is designed for a single user, where the user is charged by a fixed price for a monthly data allowance. On the other hand, a shared data plan allows a user to add extra voice/data devices to one inclusive plan and receive one bill at the end of every month. In this way, the monthly data allowance is shared by a group of users across multiple devices. Mobile operators may offer shared plans for various combinations of services, where the monthly allowances can be quotas of voice calls, SMS/email, or data. In Japan, both NTT DoCoMo and KDDI offer "Family Discount plans". NTT DoCoMo provides free email and voice calls between family members [8, 9]. KDDI provides free SMS and voice calls between family members [5]. In Singapore, StarHub offers non-shared data plans called "4G Plans" for users with the monthly allowances of data, voice calls, and SMS. The users with 4G Plans may pay extra to apply for add-on services "SharePlus", which provide up to two extra SIM cards for sharing their

Journal of Internet Services and Information Security (JISIS), volume: 6, number: 3 (August 2016), pp. 1-26
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Table 1: The *Single Line* Plan of Verizon Communications Inc. (Nov. 2014)

Monthly Data Allowance	1GB	2GB
Monthly Account Access	$60	$75
Data Overage	$15/500MB	$15/1GB

monthly allowances of data, voice calls, and SMS with their families [13]. In Hong Kong, THREE offers ”4G LTE Plan” and CSL offers ”1-FOR-ALL Service Plan”. For the 4G LTE Plan, THREE provides one primary SIM card and two secondary SIM cards. All SIM cards share the monthly allowance of voice calls and data and have unlimited SMS usage. For 1-FOR-ALL Service Plan, CSL provides one primary SIM card and two secondary SIM cards. Only the primary SIM card has voice calls, SMS, and data service. Secondary SIM cards only share the monthly allowance of data service [4, 17]. In Denmark, TDC and Telenor offer ”FamilieFordel Plan” and ”Porodični Tariff Plan” respectively for family members to share their monthly allowance of data service, and have free calls and SMS between family members [14, 15]. In Australia, Telstra offers ”Mobile Accelerate Data Share Plans” which allow users to add up to five extra SIM cards to share the monthly allowance of data service [16]. In USA, Verizon, AT&T, and Sprint offer shared data plans called ”The MORE Everything Plan”, ”Mobile Share Value Plans”, and ”Framily Plan”, respectively, which allow users to add up to ten devices to an account and share the monthly allowance of data service among all devices. Unlimited voice calls and SMS are provided by these shared plans [3, 12, 21]. Users may choose their mobile devices with service combinations, voice calls/SMS/data or data service only. The members of a group are not limited to family members.

Based on the Verizon billing plans in November 2014, we explain how to calculate the payments and analyze the relation between the payments and user behaviors. Verizon’s non-shared plan is called the Single Line Plan (the S plan) [20]. As listed in Table 1, the S plan has two data allowance options: 1 GB and 2 GB, respectively. In this table, the monthly account access field indicates the basic price, which is $60 for the 1-GB option, and $75 for the 2-GB option. If a user consumes monthly data more than the allowance, there will be an extra charge. For the 1-GB option, it is $15 per 500 MB, and $15 per 1 GB for the 2-GB option.

Verizon’s shared data plan is called The MORE Everything Plan (the M plan) that allows a group of users with up to ten devices to share an account [21]. As listed in Table 2, the M Plan offers fourteen data allowance options from 500 MB to 100 GB. The monthly account access field lists the basic prices of fourteen options; e.g., $20 for 500-MB option, $40 for 1-GB option, and so on. The number of mobile devices also affects the price. For example, if a group of users applies for the 10-GB option with five devices (three smartphones, a tablet, and an Internet device), then they should pay $230 ($80 + $40 $\times$ 3 + $10 + $20) per month. If the group consumes monthly data more than the allowance, there will be an extra charge. For the 500-MB option, the extra charge is $15 per 500 MB, and $15 per 1 GB for other options. Both the S and the M plans provide unlimited voice calls and SMS.

Verizon supports five kinds of user devices as described in Table 2 to satisfy users’ needs. Each device has a phone number associated with it even if the device cannot make a call. Smartphones are chosen by most users, which provide various features (e.g., LINE, Skype, and App Store for iOS or Google Play for Android) to access the Internet. A basic phone typically provides voice calls and text SMS. Furthermore, the users can use basic phones to connect to the Internet with mobile web and read/compose mobile emails. The basic phone has relatively limited support for third-party software in comparison to a smartphone. A tablet provides functions to access data service and is typically not designed to support

Table 2: The *MORE Everything* Plan of Verizon Communications Inc. (Nov. 2014)

Monthly Data Allowance	500MB	1 GB	2 GB	3 GB	4 GB	10 GB	15 GB	20 GB	30 GB	40 GB	50 GB	60 GB	80 GB	100 GB
Monthly Account Access	$20	$40	$50	$60	$70	$80	$100	$150	$225	$300	$375	$450	$600	$750
Data Overage	$15/ 500MB	$15/1GB												

Devices	Smartphones	Basic Phones	Tablets	Internet Devices	Connected Devices
Monthly Line Access	$40	$20	$10	$20	$5 (eg. Delphi Connect)

voice calls and SMS. An Internet device is a gateway, which allows a desktop or a laptop to connect to the Internet, such as 3G/4G USB dongles. Connected devices are gadgets that directly access to the Verizon network; for example, a Delphi Connect is a plug which allows a vehicle to connect to the Internet through an ODB-II port. User devices capable of voice calls and SMS have unlimited quotas to access these services. Note that a user can only choose the smartphone in the S plan. To simplify and strengthen our discussion, we consider smartphones as the user devices to conduct a quick and primary billing analysis based on Tables 1 and 2. Then we propose models for billing analytics with considering the user behaviors to conduct the more precise results. We note that Verizon has modified the plans several times, and the prices listed in these two tables are the version dated in November 2014. Our approach can be easily modified to accommodate other billing plans. Based on our proposed analytic model, the user can choose a more appropriate plan that reflects her/his data usage. The mobile operators can evaluate the revenue by adjusting the prices of options for shared and non-shared data plans. What a mobile operator told us is: ”as a telecom operator, we should provide appropriate billing plans to our customers to minimize their payments”. In doing so, the mobile operator is considered as a fair enterprise, and more users will select the mobile operator services. In other words, the strategy is to provide reasonable billing plans that do not make more money on individual customers, but will attract more loyal customers to select the mobile operator services to increase the total revenue.

Some existing results investigate the prices for Internet access. Molnar *et al.* [1] proposed a cost-oriented adaptive multimedia delivery mechanism to automatically download the video stream with proper quality to match the display capability of a mobile device, which reduces the prices for accessing the multimedia content. Sen *et al.* [11] discussed how to set up the pricing policy and surveyed 20 schemes of pricing policy among the static pricing and the dynamic pricing, where the static pricing schemes charge the user with a fixed monthly fee for Internet access and the offered prices of dynamics schemes vary over time. To our knowledge, no shared and non-shared data pricing analysis has been investigated in the public domain.

2 Primary Price Study

This section conducts a primary price study to answer the following question: If a group of users consumes exactly (fixed) K amount of data, then what is the best billing plan for them. This primary price study is often used by mobile operators to introduce their billing options (e.g., NTT DoCoMo [10] and THREE [18]).

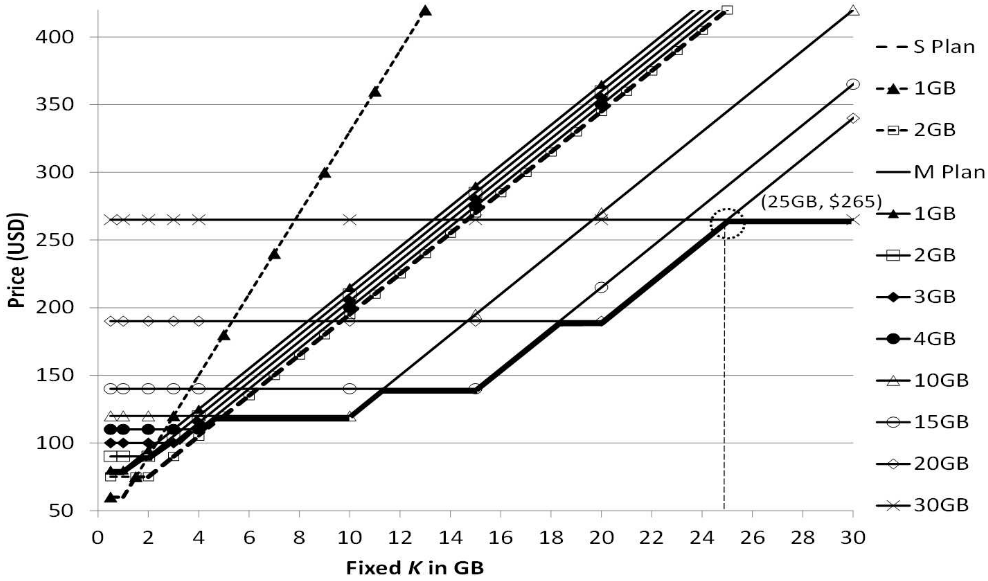


Figure 1: The primary price comparison between the S and the M plans for a single user ($I = 1$)

2.1 Price for a Single User

For a single user, Figure 1 shows the prices against fixed K for the S and the M plans with various options. There will be an extra charge if the data usage of the user exceeds the monthly data allowance described in Tables 1 and 2. The curves in Figure 1 suggest the optimal options for a single user. For example, the user is recommended to select the 20-GB option of the M plan if she/he consumes 20 - 24 GB in a month. In this case, the payment for the monthly data allowance of the 20-GB option plus the extra charge for over consumption is less than that for the 30-GB option. On the other hand, the 30-GB option of the M plan should be selected if the user consumes more than 25 GB in a month (see the dashed circle in Figure 1).

In Figure 1, the lowest prices of the M plan's options for one user with various amounts of data usage are plotted with the thick curve, and the lowest prices of the S plan's options for one user are plotted with the thick dashed curve. Figure 1 indicates that if the data usage of the user is less than 5 GB, the S plan is recommended. On the other hand, the M plan (with an appropriate option) is a better choice than the S plan if the amount of consumed data is larger than 5 GB. This result is interesting. People may think that the S plan is the right choice for a single user. But actually, the M plan with an appropriate option is a better choice for a single user with large data usage.

Figure 2 replots Figure 1 by zooming in the price curve for $0 \le K \le 6$ GB. This figure shows that if $K \le 1.5$ GB, the S plan with 1-GB option is a better choice. If $1.5 \le K \le 5$ GB, the S plan with 2-GB option is recommended. If $K \ge 5$ GB, the M plan with 10-GB option is recommended.

2.2 Price for a Group of Users

For a group of multiple users, to compare the prices between the S and the M plans, we need an extra assumption. Assume that these users separately apply for the S plan and each of the users consumes the same amount of data. That is, for a group of I users, if the total data usage is K GB, then the users select I individual S plans with the allowance $j = \frac{K}{I}$. In Figure 3, every user selects the S plan with 1-GB option or 2-GB option. For example, if the total data usage is 4 GB ($K = 4$) for two users ($I = 2$), we choose two individual S plans with 2-GB option for comparison. Figure 3 shows the prices of two users

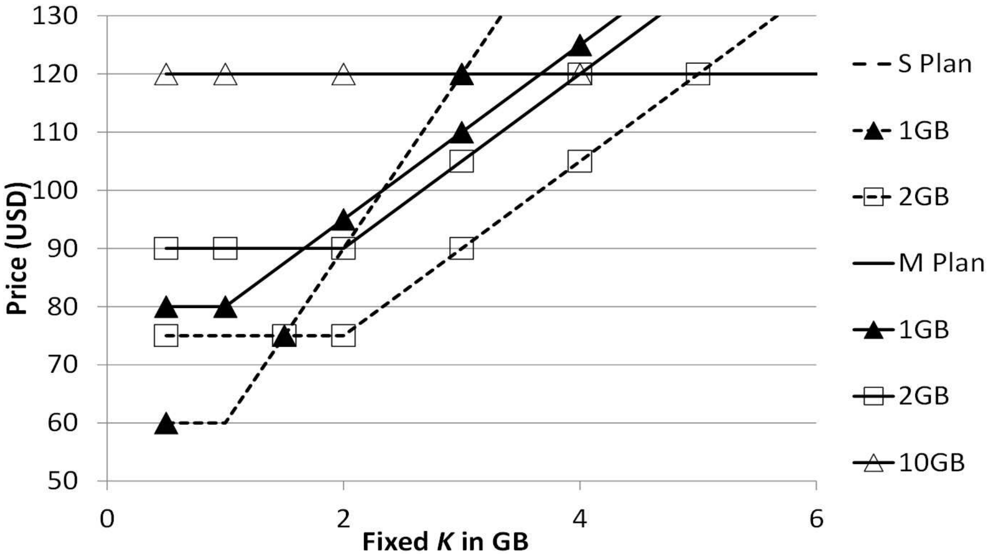


Figure 2: The primary prices of the S plan with 1-GB and 2-GB options and the M plan with 1-GB, 2-GB, and 10-GB options for $I = 1$

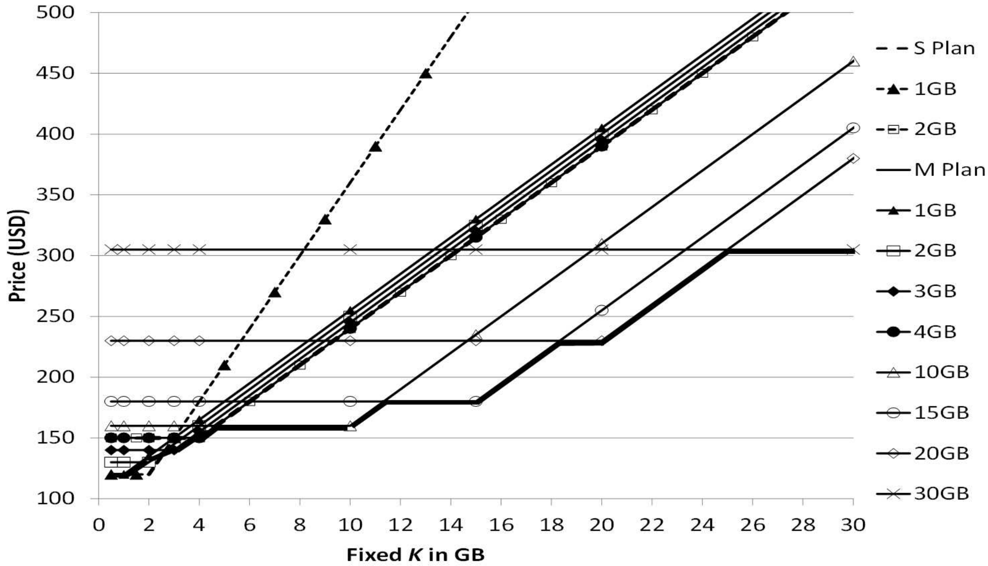


Figure 3: The primary price comparison between the S and the M plans for $I = 2$

($I = 2$) against fixed K for the S and M plans with various options. In this Figure, the lowest prices of the M plan's options are plotted with the thick curve, and the lowest prices of the S plan's options are plotted with the thick dashed curve. Figure 3 indicates that if $K \geq 2.5$ GB, the M plan with an appropriate option is better than the S plan.

We consider the price curves in Figure 3 for $0 \leq K \leq 6$ GB, and replot them in Figure 4. The figure shows that if $K \leq 2.5$ GB, the S plan with 1-GB option is a better choice. If $2.5 \leq K \leq 3.5$ GB, the M plan with 2-GB option is recommended. If $3.5 \leq K \leq 4.5$ GB, the S plan with 2-GB option and the M plan with 4-GB option are both recommended. If $K \geq 4.5$ GB, the M plan with 10-GB option (or an option with a higher monthly allowance) is a better choice.

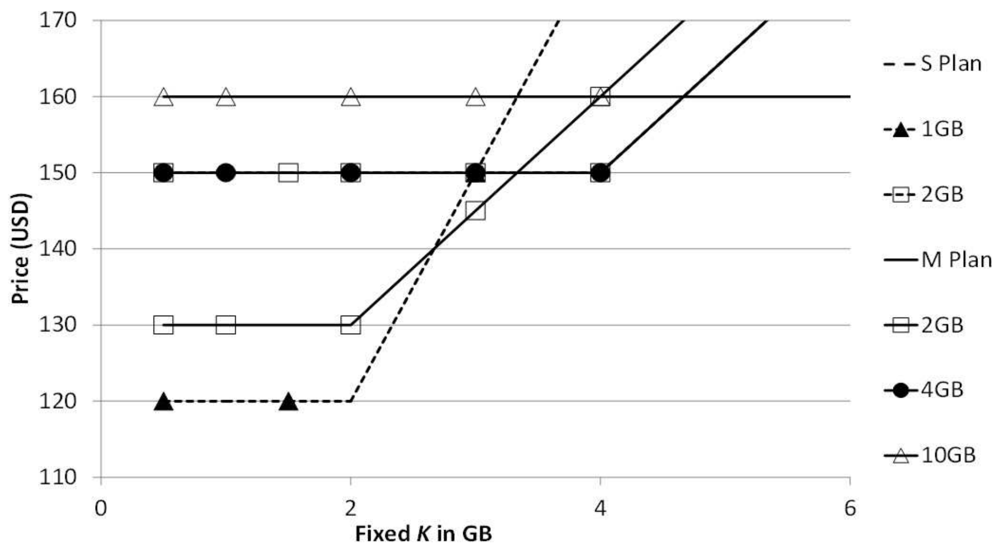


Figure 4: The primary prices of the S plan with 1-GB and 2-GB options and the M plan with 2-GB, 4-GB, and 10-GB options for $I = 2$

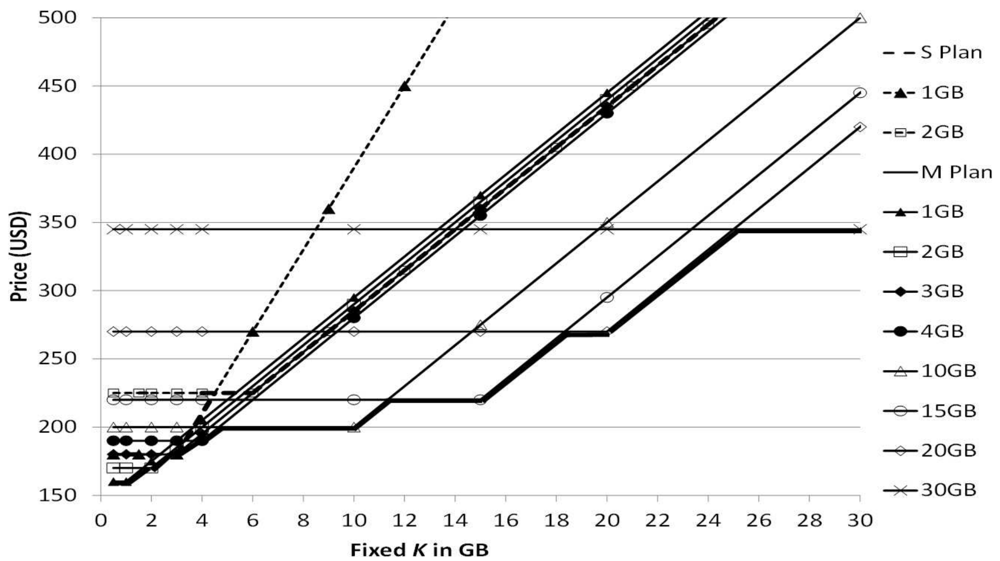


Figure 5: The primary price comparison between the S and the M plans for $I = 3$

Figure 5 shows the prices of three users ($I = 3$) against fixed K for the S and M plans with various options. In this Figure, the lowest prices of the M plan's options are plotted with the thick curve, and the lowest prices of the S plan's options are plotted with the thick dashed curve. Figure 5 indicates that the M plan with an appropriate option is better than the S plan for $I = 3$.

Figure 6 replots the price curves for $I = 3$ in the range $0 \leq K \leq 8$ GB. The figure shows that if $K \leq 3$ GB, both the S plan with 1-GB option and the M plan with 3-GB option are recommended. If $3 \leq K \leq 4.5$ GB, the M plan with 3-GB option is recommended. If $K \geq 4.5$ GB, the M plan with J-GB option ($J \geq 10$ GB) is a better choice.

The above primary price study assumes exact (fixed) data usage K for a group of users. In reality, the amounts of data usage are not fixed, and therefore should be measured from commercial mobile networks

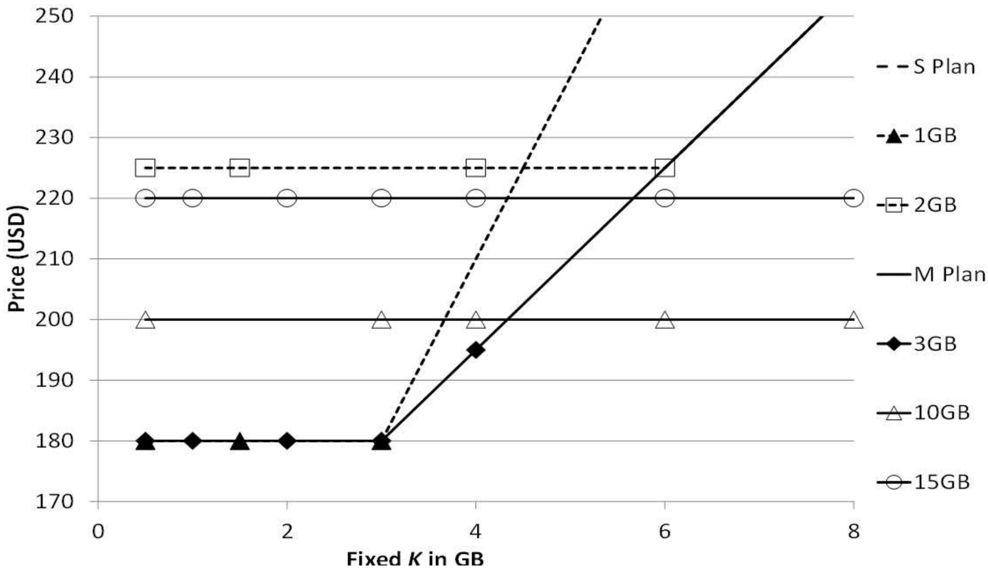


Figure 6: The primary prices of the S plan with 1-GB and 2-GB options and the M plan with 3-GB, 10-GB, and 15-GB options for $I = 3$

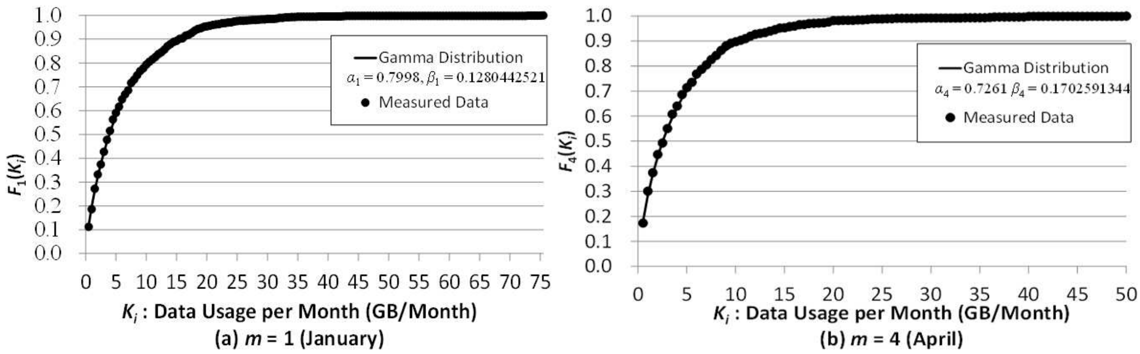


Figure 7: The data usage histograms and their Gamma distribution approximations

to reflect the user behaviors on data usage. In the next section, we derive the data usage distribution from the measured data for further analysis to find out the appropriate billing plan for a group of users.

3 Mobile Data Usage Statistics

This section investigates the statistics of mobile data usage. The data usage information was obtained from the largest telecommunications company in Taiwan. Specifically, we collected the amounts of the data consumed by 900 users per month from April 2014 to March 2015. Let K_i be the data volume consumed by user i in one month, where $1 \leq i \leq 900$. Let $\Pr[K_i]$ be the probability that user i consumes K_i GB of data in a month. Based on the measured data usage histograms for various months, Figure 7 plots the cumulative distribution functions $F_m(K_i)$ against K_i (where $m = 1$ for January and $m = 4$ for April), which is approximated by the Gamma distribution. The Gamma distribution is selected for our approximation because it can be shaped to represent many distributions as well as the measured data [6]. Let $K_i \geq 0$ and $\alpha_m, \beta_m > 0$. For the m-th month, the Gamma distribution with the shape parameter α_m

Table 3: The expected values, variances, *p*-values, and test statistics of the measured data from Chunghwa Telecom

	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09	2014/10	2014/11	2014/12	2015/01	2015/02	2015/03
$E_m[K_i]$ (GB)	4.26	4.71	5.02	5.25	5.48	5.15	5.59	5.47	5.94	6.25	5.92	6.30
V_m ($E_m[K_i]^2$)	1.639	1.420	1.582	1.596	1.317	1.250	1.148	1.135	1.160	1.392	1.327	1.247
p-value	0.7232	0.1751	0.4745	0.1640	0.4852	0.9064	0.9729	0.8586	0.9685	0.4392	0.9421	0.8929
Test statistic	0.0229	0.0366	0.0280	0.0371	0.0277	0.0187	0.0160	0.0200	0.0162	0.0287	0.0175	0.0191

and the rate parameter β_m has the probability density function

$$f_m(K_i) = \frac{{\beta_m}^{\alpha_m} {K_i}^{\alpha_m - 1} e^{-\beta_m K_i}}{\Gamma(\alpha_m)} \tag{1}$$

where $\Gamma(\alpha_m) = \int_{\tau=0}^{\infty} e^{-\tau} \tau^{\alpha_m - 1} d\tau$. In (1), the mean E$m[K_i]$ = $\frac{\alpha_m}{\beta_m}$ and the variance V$m[K_i] = \frac{\alpha_m}{{\beta_m}^2}$. The cumulative distribution function for the *m*-th month is $F_m(K_i) = \int_0^{K_i} f_m(K_i) dk$. In Figure 7 (a), the data usage of January (the • dots) is approximated by the Gamma distribution with $E_1[K_i]$ = 6.25 GB and $V_1[K_i]$ = 1.392 $E_1[K_i]^2$ (the solid cure). The data usage of April is approximated by the Gamma distribution with $E_4[K_i]$ = 4.26 GB and $V_4[K_i]$ = 1.639 $E_4[K_i]^2$ (Figure 7 (b)). Table 3 shows the expected values, variances, *p*-values, and test statistics of $F_m(K_i)$ for individual months.

The approximations are validated by the Kolmogorov-Smirnov (K-S) test, which is a nonparametric test to compare a sample dataset with a hypothesized continuous distribution. The null distribution of this statistic is calculated under the null hypothesis where the samples are drawn from the reference distribution. The null hypothesis states that the measured data samples of Chunghwa Telecom Co., Ltd. (CHT) came from a population of a Gamma distribution. In this test, a *p*-value is the probability of obtaining the measured sample results when the null hypothesis is true. The hypothesis regarding the distributional form is rejected if the *p*-value calculated based on the test is less than a chosen significance level (e.g., 0.05 in our example). On the other hand, a *p*-value greater than the significance level means that the null hypothesis has higher probability to be true and the null hypothesis will be accepted. The *p*-values for the consecutive 12 months in the observation period are 0.4392, 0.9421, 0.8929, 0.7232, 0.1751, 0.4745, 0.1640, 0.4852, 0.9064, 0.9729, 0.8586, and 0.9685, respectively, which are much larger than 0.05. From another point of view, we do not reject the null hypothesis at the significance level 0.05 if the test statistic is less than a critical value, where the test statistic is the maximum vertical deviation between the curve of the measured data and the curve of the Gamma distribution, and the critical value is the acceptable maximum vertical deviation between the two curves. From the table "Critical values for the Kolmogorov-Smirnov test for goodness of fit" [7], the critical value is 0.0453 when the number of data samples is 900 at the significance level 0.05. Since the test statistics for the consecutive 12 months in Table 3 are less than the critical value 0.0453 at the 0.05 level, there is insufficient evidence to reject the assumption that the data are drawn from a specific Gamma distribution. In other words, the K-S test at a significance level of 0.05 shows that the measured data usage of 900 users fits the Gamma distribution.

4 Analytic Modeling

This section proposes an analytic model for the M and the S plans using the approximated distributions of CHT measured data. To make a fair comparison, we assume that a group of users consumes the same amounts of data for both the M and the S plans. Consider a group of I users. Let K_i be the data volume consumed by user i in one month. Assume that for $1 \leq i \leq I$, K_i are independent and identically distributed (i.i.d.) random variables with the density functions $f_m(K_i)$ for the m-th month, where $1 \leq m \leq 12$. These $f_m(K_i)$ functions are approximated from the measured data of CHT in the previous section. Let K be the total data usage consumed by the group of I users in one month, that is $K = \sum_{i=1}^{I} K_i$. Let $g_m(K)$ be the density function that I users consume K GB of data in the m-th month. For $I = 1$, $g_m(K) = f_m(K)$. For $I = 2$, $g_m(K) = \int_{K_1=0}^{K} f_m(K_1) f_m(K-K_1) dK_1$. For $I \geq 3$,

$$g_m(K) = \int_{K_1=0}^{K} \int_{K_2=0}^{K-K_1} \cdots \int_{K_{I-1}=0}^{K-K_1 \cdots K_{I-2}} \prod_{i=1}^{I-1} f_m(K_i) f_m(K - K_1 \cdots - K_{I-1}) dK_{I-1} \cdots dK_2 dK_1 \tag{2}$$

To simplify our discussion, we assume that each of I users selects the same monthly data allowance j of the S plan. In the M plan, the group has the monthly data allowance of J GB, where $J = j \times I$. If $K > J$, there is an additional payment for extra data usage of (K–J) GB. Consider u GB of data (e.g., u = 0.5 or 1). Let $L_u(K) = \lceil \frac{K}{u} \rceil \times u$ GB. That is, $L_u(K)$ is the smallest multiple of u GB, which is greater than or equal to K.

For the M plan, let $C_{M,I,J,m}$ be the expected payment of the m-th month for the I-user group with a monthly data allowance of J GB. Let $c_{M,I,J}$ be the monthly payment for the M plan with J-GB option without overcharge. Let $o_{M,I,J}$ be the extra payment per u GB if the data usage of I users exceeds J GB (i.e., their monthly data allowance). The parameter u of $L_u(K)$ is dependent on the option of the M plan. In Table 2, the extra payment is \$15 per 500 MB for the 500-MB option, and \$15 per 1 GB for other options. Therefore, u = 0.5 for the 500-MB option, and u = 1 for other options in the M plan. Let

$$N_m = \frac{\max\left[L_u(K) - J, 0\right]}{u} \tag{3}$$

for the m-th month. Then the I-user group pays extra $N_m \times o_{M,I,J}$ dollars for the m-th month in the M plan. For example, if the I-user group applies for the 10-GB option and consumes 11.5 GB in a month (i.e., $J = 10$ and $K = 11.5$), the payment is calculated as follows: the I-user group is charged by the monthly payment $c_{M,I,J} = 100 + 40 \times I$ dollars for 10-GB option. For $K = 11.5$, $u = 1$ and $L_u(K) = 12$. Since $J = 10$, from (3), the users are charged for extra 2 GB. In this case, the extra payment is $\left(\frac{12-10}{1}\right) \times o_{M,I,J}$, where $o_{M,I,J} = 15$ dollars. Thus the final charge for the month is $100 + 40 \times I + (12–10) \times 15$ dollars.

Let $\Pr[N_m = n]$ be the probability that the I-user group pays extra $n \times o_{M,I,J}$ dollars for the m-th month in the M plan. Then from (2) and (3)

$$\Pr[N_m = n] = \Pr[(n-1)u + J < K \leq nu + J] = \int_{K=(n-1)u+J}^{nu+J} g_m(K) dK \tag{4}$$

From (4), we have

$$\begin{aligned} C_{M,I,J,m} &= \sum_{n=0}^{\infty} \left\{ c_{M,I,J} + n \times o_{M,I,J} \Pr[N_m = n] \right\} \\ &= c_{M,I,J} + o_{M,I,J} \left\{ \sum_{n=0}^{\infty} n \Pr[N_m = n] \right\} \\ &= c_{M,I,J} + o_{M,I,J} \left[\sum_{n=0}^{\infty} n \int_{K=(n-1)u+J}^{nu+J} g_m(K) dK \right] \end{aligned} \tag{5}$$

For user i, if $\alpha_m = 1$ in $f_m(K_i)$, i.e., the variance $\mathrm{V}_m = \mathrm{E}_m[K_i]^2$, then K_i are exponentially distributed, and K has the Erlang probability density function with the shape parameter I and the scale parameter

$\beta_m = I/\mathrm{E}_m[K]$:

$$g_m(K) = \frac{\beta_m^I K^{I-1} e^{-\beta_m K}}{(I-1)\,!} \text{ for } K, I \geq 1 \tag{6}$$

Substitute (6) into (4) to yield

$$\begin{aligned}
\Pr[N_m = n] &= \int_{K=(n-1)u+J}^{nu+J} g_m(K)dK \\
&= \int_{K=(n-1)u+J}^{nu+J} \left[\frac{{\beta_m}^I K^{I-1} e^{-\beta_m K}}{(I-1)\,!}\right] dK \\
&= 1 - \sum_{i=0}^{I-1} \left[\frac{e^{-\beta_m K}(\beta_m K)^i}{i!}\right] \Bigg|_{K=(n-1)u+J}^{nu+J}
\end{aligned} \tag{7}$$

Let $Z_m(i) = \frac{\beta_m^i}{i!}$, and (7) is re-written as

$$\begin{aligned}
\Pr[N_m = n] &= 1 - \sum_{i=0}^{I-1} Z_m(i) K^i e^{-\beta_m K} \Bigg|_{K=(n-1)u+J}^{nu+J} \\
&= \sum_{i=0}^{I-1} Z_m(i) \left\{ [(n-1)u+J]^i e^{-\beta_m[(n-1)u+J]} - (nu+J)^i e^{-\beta_m(nu+J)} \right\} \\
&= \sum_{i=0}^{I-1} Z_m(i) e^{-\beta_m J} \left\{ [(n-1)u+J]^i e^{-\beta_m u(n-1)} - (nu+J)^i e^{-\beta_m un} \right\}
\end{aligned} \tag{8}$$

From (8), we have

$$\begin{aligned}
\sum_{n=0}^{\infty} n \Pr[N_m = n] &= \sum_{i=0}^{I-1} Z_m(i) e^{-\beta_m J} \left\{ \sum_{n=0}^{\infty} n \left\{ [(n-1)u+J]^i e^{-\beta_m u(n-1)} - (nu+J)^i e^{-\beta_m un} \right\} \right\} \\
&= \sum_{i=0}^{I-1} Z_m(i) e^{-\beta_m J} \left[J^i + \sum_{n=0}^{\infty} (nu+J)^i e^{-\beta_m un} \right] \\
&= \sum_{i=0}^{I-1} Z_m(i) e^{-\beta_m J} \left[J^i + \sum_{n=1}^{\infty} \sum_{l=0}^{i} \binom{i}{l} J^{i-l} (nu)^l e^{-\beta_m un} \right] \\
&= \sum_{i=0}^{I-1} Z_m(i) e^{-\beta_m J} \left[J^i + \sum_{l=0}^{i} \binom{i}{l} J^{i-l} u^l \left(\sum_{n=1}^{\infty} n^l e^{-\beta_m un} \right) \right] && (9) \\
&\approx \sum_{i=0}^{I-1} Z_m(i) e^{-\beta_m J} \left[J^i + \sum_{l=0}^{i} \binom{i}{l} J^{i-l} u^l \left(\int_{t=0}^{\infty} t^l e^{-\beta_m ut} dt \right) \right] && (10) \\
&= \sum_{i=0}^{I-1} Z_m(i) e^{-\beta_m J} \left\{ J^i + \sum_{l=0}^{i} \binom{i}{l} J^{i-l} u^l \left[\frac{l!}{(\beta_m u)^{l+1}} \right] \right\} \\
&= \sum_{i=0}^{I-1} (\beta_m J)^i e^{-\beta_m J} \left[\frac{1}{i!} + \sum_{l=0}^{i} \frac{1}{(i-l)!(\beta_m J)^l \beta_m u} \right] && (11)
\end{aligned}$$

Substitute (11) into (5) to yield

$$C_{M,I,J,m} = c_{M,I,J} + o_{M,I,J} \left\{ \sum_{i=0}^{I-1} \frac{(\beta_m J)^i e^{-\beta_m J}}{i!} + \sum_{i=0}^{I-1} \sum_{l=0}^{i} \left[\frac{(\beta_m J)^l e^{-\beta_m J}}{l!} \right] \left(\frac{1}{\beta_m u} \right) \right\} \tag{12}$$

If $I = 1$, we have

$$C_{M,1,J,m} = c_{M,1,J} + o_{M,1,J}\left[e^{-\beta_m J}\left(1+\frac{1}{\beta_m u}\right)\right] \tag{13}$$

For a group of I users who apply for the S plan with j-GB option, we have $J = j \times I$. Let $C_{S,I,J,m}$ be the expected net payment of the m-th month for these I users. There will be an extra payment per GB if $K_i > j$. Let $c_{S,j}$ be the monthly payment of the S plan with j-GB option without overcharge. Let $o_{S,j}$ be the extra payment per u GB if the data usage of user i exceeds her/his monthly data allowance of j GB. Like the M plan, the parameter u of $L_u(K_i)$ is dependent on the option of the S plan. In Table 1, the extra payment is \$15 per 500 MB for the 1-GB option, and \$15 per 1 GB for the 2-GB option. Therefore, $u = 0.5$ for the 1-GB option, and $u = 1$ for the 2-GB option in the S plan. For user i, (3) can be modified as $N_{m,i} = \frac{\max[L_u(K_i)-j,0]}{u}$, and user i pays extra $N_{m,i} \times o_{S,j}$ dollars for the m-th month in the S plan. For example, if user i applies for the 1-GB option and consumes 2.3 GB in a month (i.e., $j = 1$ and $K_i = 2.3$), the payment is calculated as follows: for the 1-GB option, $c_{S,j} = 60$ dollars. For $K = 2.3$ and $u = 0.5$, the user is charged for extra 1.5 GB since $L_u(K_i)$ is 2.5 and $j = 1$. In this case, the extra payment is $\left(\frac{2.5-1}{0.5}\right) \times o_{S,j}$, where $o_{S,j} = 15$ dollars. Therefore, the final payment is $60 + \left(\frac{2.5-1}{0.5}\right) \times 15$ dollars.

Let $\Pr[N_{m,i} = n_i]$ be the probability that user i pays extra $n_i \times o_{S,j}$ dollars for the m-th month in the S plan with j-GB option. Then from (4), we have

$$\begin{aligned}\Pr[N_{m,i} = n_i] &= \Pr[(n_i-1)u + j < K_i \le n_i u + j] \\ &= \int_{K_i=(n_i-1)u+j}^{n_i u+j} f_m(K_i)dK_i\end{aligned} \tag{14}$$

and

$$\begin{aligned}C_{S,I,J,m} &= \sum_{i=1}^{I}\sum_{n_i=0}^{\infty}\left\{c_{S,j} + n_i o_{S,j}\Pr[N_{m,i} = n_i]\right\} \\ &= \sum_{i=1}^{I}\left\{c_{S,j} + o_{S,j}\left\{\sum_{n_i=0}^{\infty} n_i \Pr[N_{m,i} = n_i]\right\}\right\} \\ &= \sum_{i=1}^{I}\left\{c_{S,j} + o_{S,j}\left[\sum_{n_i=0}^{\infty} n_i \int_{K_i=(n_i-1)u+j}^{n_i u+j} f_m(K_i)dK_i\right]\right\}\end{aligned} \tag{15}$$

For user i, if $\alpha_m = 1$ in $f_m(K_i)$, i.e., the variance $\mathrm{V}_m = \mathrm{E}_m[K_i]^2$, then K_i are exponentially distributed for the m-th month. That is, the data usage K_i has the Exponential probability density function with the scale parameter β_m:

$$f_m(K_i) = \beta_m e^{-\beta_m K_i} \text{ for } K_i \ge 0 \tag{16}$$

Substitute (16) into (14) to yield

$$\begin{aligned}\Pr[N_{m,i} = n_i] &= \Pr[(n_i-1)u + j < K_i \le n_i u + j] \\ &= e^{-\beta_m j}\left[e^{-\beta_m u(n_i-1)} - e^{-\beta_m u n_i}\right]\end{aligned} \tag{17}$$

From (17), we have

$$\begin{aligned}\sum_{n_i=0}^{\infty} n_i \Pr[N_{m,i} = n_i] &= e^{-\beta_m j}\left\{\sum_{n_i=0}^{\infty} n_i\left[e^{-\beta_m u(n_i-1)} - e^{-\beta_m u n_i}\right]\right\} \\ &= e^{-\beta_m j}\left(1+\frac{1}{\beta_m u}\right)\end{aligned} \tag{18}$$

Table 4: The scaled-down $\mathrm{E}_m[K_i]$ for $\mathrm{E}[K_i]$ = 0.5, 0.7, 1, 2, 3, 4 GB

$\mathrm{E}[K_i]$ (GB)	$\mathrm{E}_m[K_i]$ (GB)											
	2014/04	2014/05	2014/06	2014/07	2014/08	2014/09	2014/10	2014/11	2014/12	2015/01	2015/02	2015/03
0.5	0.391	0.433	0.461	0.482	0.503	0.473	0.513	0.502	0.545	0.574	0.544	0.579
0.7	0.548	0.606	0.645	0.675	0.705	0.662	0.719	0.703	0.764	0.803	0.761	0.810
1	0.782	0.865	0.922	0.964	1.006	0.946	1.027	1.005	1.091	1.148	1.087	1.157
2	1.565	1.730	1.844	1.928	2.013	1.892	2.053	2.009	2.182	2.296	2.174	2.314
3	2.347	2.595	2.766	2.893	3.019	2.837	3.080	3.014	3.273	3.444	3.262	3.471
4	3.129	3.460	3.688	3.857	4.026	3.783	4.107	4.018	4.364	4.591	4.349	4.628
5.445	4.26	4.71	5.02	5.25	5.48	5.15	5.59	5.47	5.94	6.25	5.92	6.30

Substitute (18) into (15) to yield

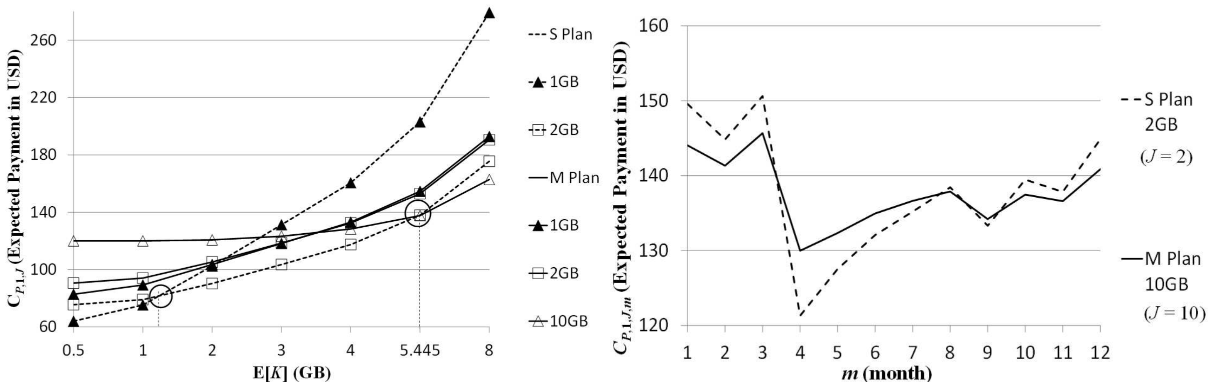
$$\begin{aligned} C_{S,I,J,m} &= \sum_{i=1}^{I}\sum_{n_i=0}^{\infty}\left\{c_{S,j}+n_i o_{S,j}\Pr[N_{m,i}=n_i]\right\} \\ &= I\left\{c_{S,j}+o_{S,j}\left[e^{-\beta_m j}\left(1+\frac{1}{\beta_m u}\right)\right]\right\} \end{aligned} \tag{19}$$

We have conducted simulation experiments to validate against (5) for Gamma $f_m(K_i)$, (12) for Erlang $f_m(K_i)$, and (9). The errors are within 1% for (5), 1% for (9), and 3% for (12). The error for (12) is higher than those for (5) and (9) (but is still small) due to the approximation at (10). The above results show that both analytic analysis and simulation experiments are consistent.

5 Comparing the Verizon's S and M Plans

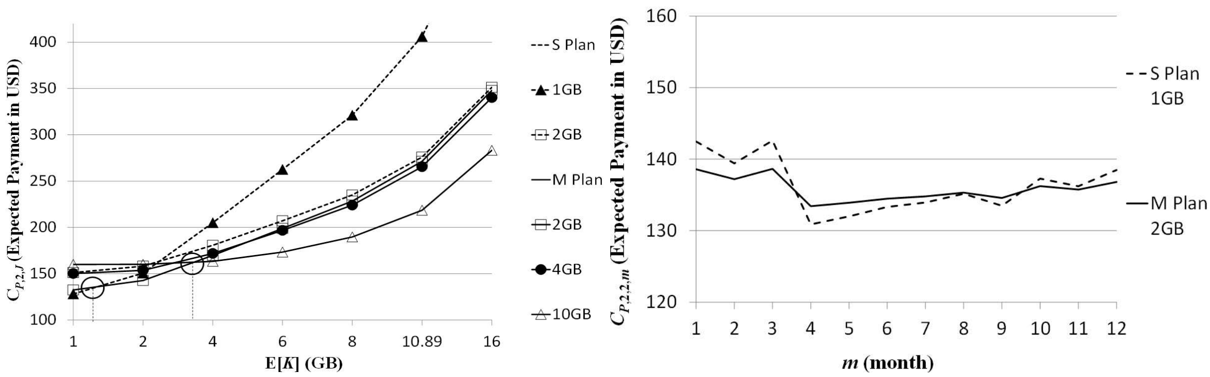
This section compares the S and the M plans based on the analytic model developed in the previous section. In our analysis, the approximated monthly distributions of the measured data of CHT derived in Section 3 are used as the inputs. Let $\mathrm{E}[K_i]$ be the average amount of data consumed by user i in a month, then $\mathrm{E}[K_i]=\left(\frac{1}{12}\right)\sum_{m=1}^{12}\mathrm{E}_m\left[K_i\right]$. Let $\mathrm{E}[K]$ be the average amount of data consumed by a group of I users in a month. Then $\mathrm{E}[K]=\mathrm{E}\left[\sum_{i=1}^{I}K_i\right]$. For the data measured from Chunghwa Telecom, $\mathrm{E}[K_i]=5.445$ GB. To further investigate the S and the M plans for different user behaviors in terms of the average amounts of data usage, we also consider the scenarios where $0.5\leq \mathrm{E}[K_i]\leq 8$ GB. In these scenarios, we assume that the data usage of a user has the Gamma distribution with the same variance as that measured from CHT; that is, the variance of the data usage distribution for every month is the same as that for the measured data of CHT usage but $\mathrm{E}_m[K_i]$ is scaled down or up as $(\mathrm{E}[K_i]/5.445)\times\mathrm{E}_m[K_i]$. Table 4 lists the scaled-down $\mathrm{E}_m[K_i]$ for various $\mathrm{E}[K_i]$ scenarios. Denote P as a billing plan. For example, $P=S$ is the non-shared data plan and $P=M$ is the shared data plan for Verizon. Let $C_{P,I,J,m}$ be the expected payment of the m-th month for plan P with I users and monthly data allowance of J GB. Let $C_{P,I,J}=\left(\frac{1}{12}\right)\sum_{m=1}^{12}C_{P,I,J,m}$, which is the expected payment for a month. For a group of I users, to make a reasonable comparison between the M plan with J-GB option and the S plan with j-GB option, we assume that in the S plan, I users individually apply for the S plan with the monthly data allowance $j=\frac{J}{I}$, That is,

$$C_{S,I,J}=I\times C_{S,1,j}\text{ for }j=\frac{J}{I} \tag{20}$$



(a) (b)

Figure 8: The expected payments of the S and the M plans for $I = 1$. (a) $C_{P,1,J}$ for $J = 1,2,10$ GB; (b) $C_{P,1,J,m}$ with E[K_i]=5.445 GB



(a) (b)

Figure 9: The expected payments of the S and the M plans for $I = 2$. (a) $C_{P,2,J}$ for $J = 1,2,4,10$ GB; (b) $C_{P,2,2,m}$ with E[K_i]=0.7 GB

For $I = 1$, the expected payments $C_{P,1,J}$ of the S and the M plans are plotted in Figure 8 (a). The dashed curves represent the payments for the S plan and the solid curves represent the payments for the M plan. The ▲ curves illustrate the expected payments of the S and the M plans with 1-GB option. The □ curves illustrate the expected payments of the S and the M plans with 2-GB option. The △ curve illustrates the expected payments of the M plan with 10-GB option. This figure shows that if $\mathrm{E}[K] \leq 1.2$ GB, the S plan with 1-GB option is recommended. If $1.2 \leq \mathrm{E}[K] \leq 5.445$ GB, the S plan with 2-GB option is a better choice. If $\mathrm{E}[K] \geq 5.445$ GB, the M plan with 10-GB option is recommended. The results of Figure 8 (a) are similar to what we observed in the primary price study in Figure 2. Figure 8 (b) plots for the M plan with 10-GB option (the solid curve) and the S plan with 2-GB option (the dashed curve). The curves show that although the user of the S plan pays more for the entire year, he/she pays less from April to September.

For $I = 2$, the expected payments $C_{P,2,J}$ of the S and the M plans are plotted in Figure 9 (a). This figure shows that if $\mathrm{E}[K] \leq 1.3$ GB, the S plan with 1-GB option is recommended. If $1.3 \leq \mathrm{E}[K] \leq 3.5$ GB, the M plan with 2-GB is a better choice. If $\mathrm{E}[K] \geq 3.5$ GB, the M plan with 10-GB option is recommended. Figure 9 (b) illustrates the $C_{P,2,2,m}$ curves for the S plan with 1-GB option and the M plan with 2-GB option (i.e., $j = 1$ GB and $J = 2$ GB). The figure indicates that although the two users of the

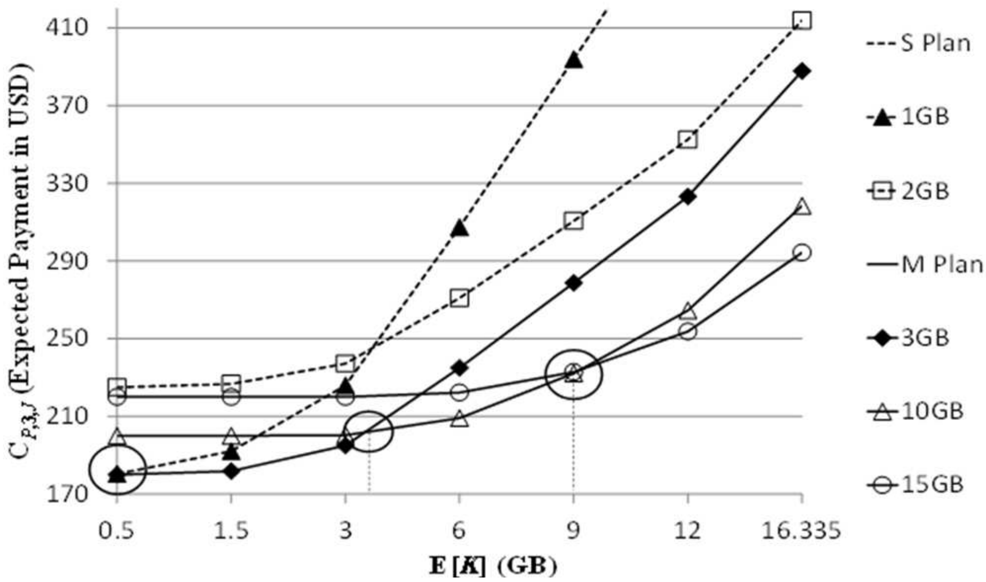


Figure 10: The expected payment of the S and the M plans for $I = 3$

S plan pay more for the entire year, they pay less from April to September. Compared with the primary price study in Figure 4, Figure 9 provides more accurate results.

In Figure 10, the expected payments $C_{P,3,J}$ for $P = S$ and M, where $I = 3$. This figure shows that the M plan is better than the S plan for $\mathrm{E}[K] > 0.5$ GB. For $\mathrm{E}[K] \leq 0.5$, both the M plan with 3-GB option and the S plan with 1-GB option are recommended. If $0.5 \leq \mathrm{E}[K] \leq 3.5$ GB, the M plan with 3-GB option is recommended. If $3.5 \leq \mathrm{E}[K] \leq 9$, the M plan with 10-GB option is a better choice. If $\mathrm{E}[K] \geq 9$ GB, the M plan with 15-GB option is recommended.

The results of Figure 10 are different from what we observed in the primary price study in Figure 6. Clearly, the results in Figures 9 and 10 are more accurate than those of the primary price study in Section 2.

6 Comparing Verizon's Limited and Unlimited Plans

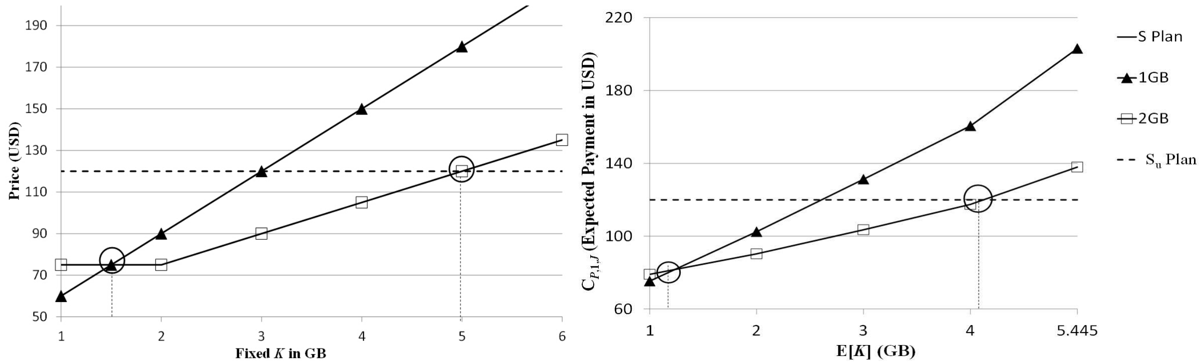
This section compares the S and the M plans with Verizon's all-you-can-eat (unlimited data service) plans (note that Verizon has cancelled the unlimited plans). Verizon's non-shared plan with unlimited data service is also called the *Single-Line* Plan. For the discussion purposes, we call it the *Unlimited Single Line* Plan (the S_u plan). The shared plan with unlimited data service is called the *Family Share* Plan, and we call it the *Unlimited MORE Everything* Plan (the M_u plan) [19]. As listed in Table 5, the monthly account access fee of the S_u plan is \$120 with unlimited voice call/SMS/data service. The M_u plan is a service for a group of users with up to five lines, where the monthly account access fee is \$180 for the first two lines with unlimited voice call/SMS/data service, and for the third, the fourth, and the fifth lines, each of additional lines is charged for extra \$50 (i.e., \$230 for three users, \$280 for four users, and \$330 for five users).

6.1 Comparing the S_u and the S Plans

For a single user ($I = 1$), the primary price study is conducted to provide a quick comparison between the S and the S_u plans in Figure 11 (a). The solid cures illustrate the payments of the S plan and the dashed

Table 5: The *Unlimited Single Line* Plan and the *Unlimited MORE Everything* Plan of Verizon Communications Inc. (Jul. 2011)

Plan	Unlimited Single-Line (S_u)	Unlimited More Everything (M_u)
Monthly Account Access	$120	$180 (for first two lines)
TALK&TEXT&DATA	UNLIMITED	UNLIMITED
Additional lines	N/A	$50 (up to three additional lines)



(a) (b)

Figure 11: Comparing the S and the Su plans for $I = 1$. (a) The primary price study; (b) The expected payments ($C_{P,1,J}$)

line represents the fixed payment of the S_u plan (which is $120). In this case, $K = K_1$. Also, since the K value is assumed to be fixed in the primary price study, $K = \text{E}[K]$. In Figure 11 (a), if $K \le 1.5$ GB, the S plan with 1-GB option is recommended. If $1.5 \le K \le 5$ GB, the S plan with 2-GB is a better choice. If $K \ge 5$ GB, the S_u plan is recommended. From the measured data of CHT and the analysis in Section 4, the expected payments $C_{P,1,J}$ of the S_u and the S plans with various E[K] are plotted in Figure 11 (b). In this figure, if $\text{E}[K] \le 1.2$ GB, the S plan with 1-GB option is recommended. For $1.2 \le \text{E}[K] \le 4.1$ GB, the S plan with 2-GB option is a better choice. If $\text{E}[K] \ge 4.1$ GB, the S_u plan is recommended. Clearly, the results in Figure 11 (b) are more accurate than that in Figure 11 (a), and the primary price study only provides quick and rough billing plan suggestions. Figure 12 plots $C_{S,1,1,m}$ for $\text{E}[K] = 3$ GB and $C_{S,1,2,m}$ for $\text{E}[K] = 4$ GB. For $\text{E}[K] = 3$ GB, the expected payment of the S plan with 1-GB option ($C_{S,1,1}$ = $131.26) is higher than that of the S_u plan ($C_{S_u,1}$ = $120). For $\text{E}[K] = 4$ GB, the expected payment of the S plan with 2-GB option ($C_{S,1,2}$ = $117.46) is lower than that of the S_u plan. Note that when comparing these billing plans, the user may mistakenly select a plan such that he/she pays less for some months but pays more for the entire year.

6.2 Comparing the M_u and the M Plans

Figure 13 (a) shows the results of the primary price study for the M and the Mu plans with $I = 2$. The solid lines illustrate the payments of the M plan and the dashed line represents the fixed payment of the M_u plan (which is $180). The figure shows that if $K \le 4$ GB, the M plan with 2-GB option is a better choice for the users. If $4 \le K \le 11.5$ GB, the M plan with 10-GB option is recommended. If $11.5 < K \le 15$ GB, both the M plan with 15-GB option and the M_u plan are recommended. If $K \ge 15$ GB, the M_u plan is a better choice. Based on CHT's measured data, Figure 13 (b) shows the expected payments $C_{P,2,J}$ against

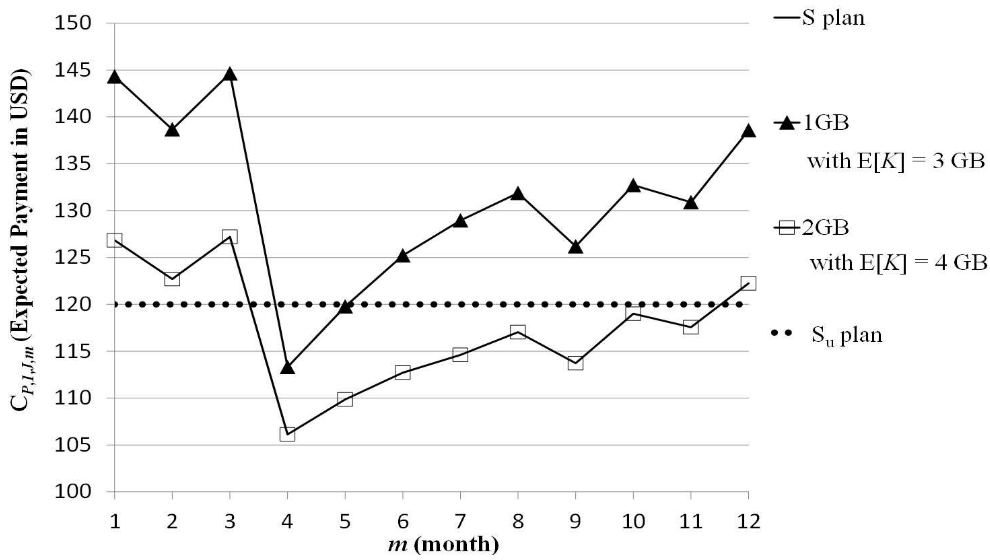
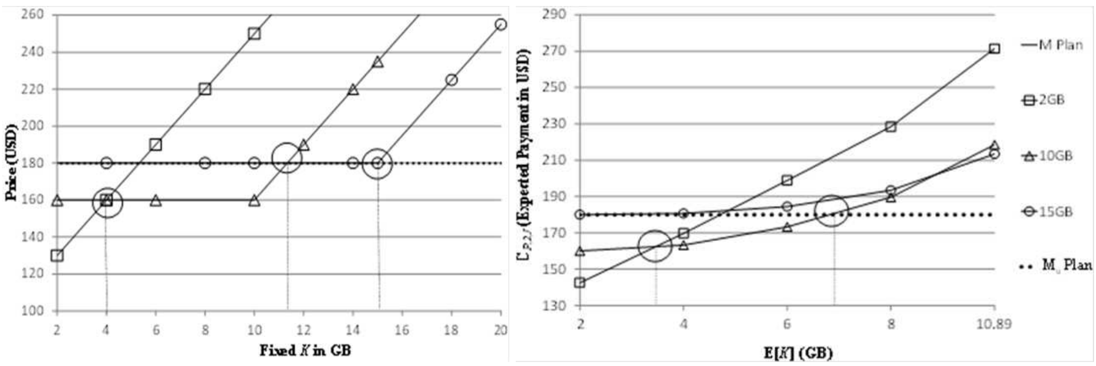


Figure 12: Comparing the S plan with 1-GB and 2-GB options and the S_u plan ($I = 1$, E[K] = 4 GB)



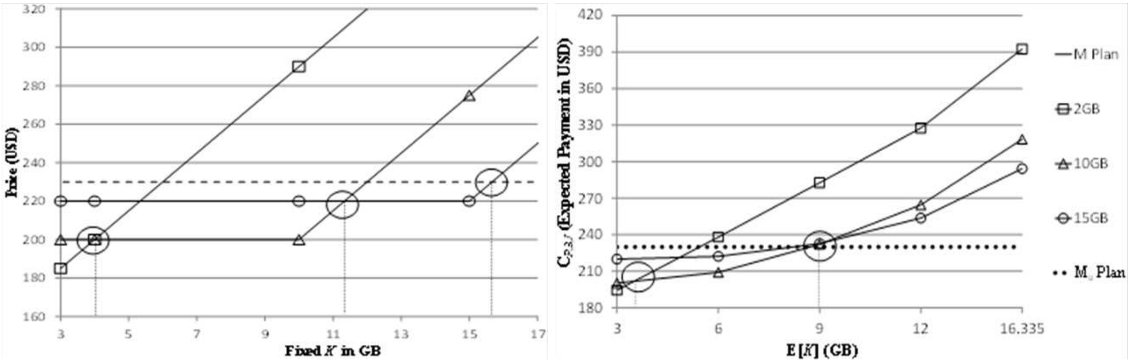
(a) (b)

Figure 13: Comparing the M_u plan and the M plan with 2-GB, 10-GB, and 15-GB options for $I = 2$. (a) The primary price study; (b) The expected payments ($C_{P,2,J}$)

E[K] for the M_u plan ($P = M_u$) and the M plan ($P = M$) with various options. Figure 13 (b) shows that if E[K] $\leq$ 3.5 GB, the M plan with 2-GB option is a better choice. If 3.5 $\leq$ E[K] $\leq$ 7 GB, the M plan with 10-GB option is recommended. If E[K] $\geq$ 7 GB, the M_u plan is recommended.

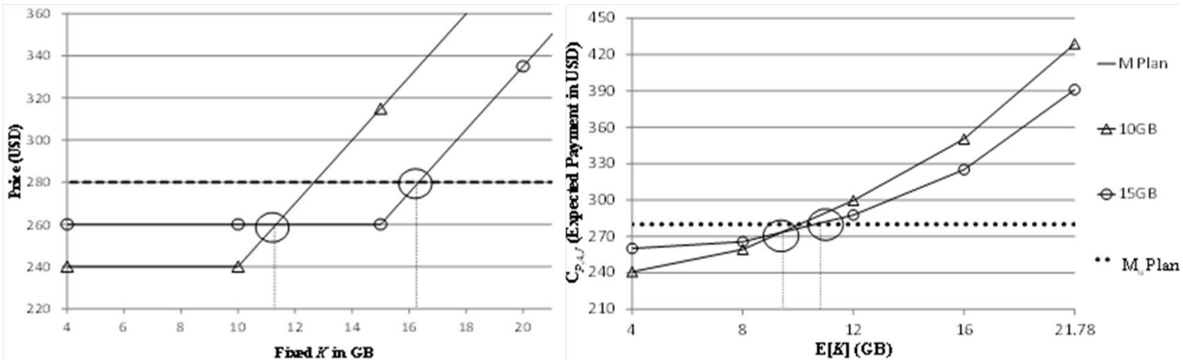
For $I = 3$, Figure 14 (a) shows the results of the primary price study, where the dashed line is the fixed payment of the M_u plan (which is $230). The figure indicates that if $K \leq 4$ GB, the M plan with 2-GB option is recommended. If $4 \leq K \leq 11.5$ GB, the M plan with 10-GB option is a better choice. If $11.5 < K \leq 15.5$ GB, the M plan with 15-GB option is recommended. If $K > 15.5$ GB, the M_u plan is recommended. By considering the user behaviors from the measured data of CHT, Figure 14 (b) shows the expected payments $C_{P,3,J}$ against E[K] for the M_u plan ($P = Mu$) and the M plan ($P = M$). In Figure 14 (b), if E[K] $\leq$ 3.5 GB, the M plan with 2-GB option is a better choice. If 3.5 $\leq$ E[K] $\leq$ 9 GB, the M plan with 10-GB option is recommended. If E[K] $\geq$ 9 GB, the M_u plan is recommended.

For $I = 4$, the price curves in Figure 15 (a) show that if $K \leq 11$ GB, the M plan with 10-GB option is recommended. If $11 \leq K \leq 16.5$ GB, the M plan with 15-GB option is recommended. If $K \geq 16.5$ GB, the M_u plan is recommended. In this figure, the solid lines are the payments of the M plans ($P = M$).



(a) (b)

Figure 14: Comparing the expected payments of the Mu plan and the M plan with 2-GB, 10-GB, and 15-GB options for $I = 3$. (a) The primary price study; (b) The expected payments ($C_{P,3,J}$)



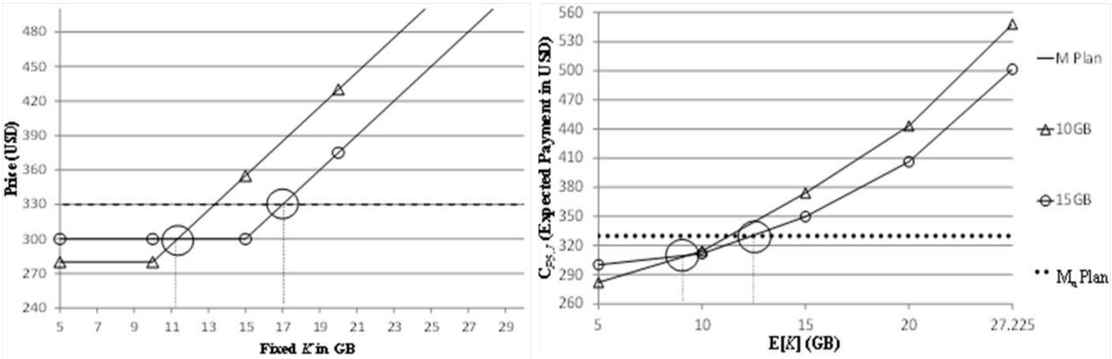
(a) (b)

Figure 15: Comparing the Mu plan and the M plan with options 10-GB and 15-GB options for $I = 4$. (a) The primary price study; (b) The expected payments ($C_{P,4,J}$)

The dashed line is the fixed payment of the M_u plan ($P = Mu$) (which is \$280). Figure 15 (b) shows the expected payments $C_{P,4,J}$ against $E[K]$ for the M_u plan and the M plan with various options. In this figure, if $E[K] \leq 9.5$ GB, the M plan with 10-GB option is recommended. If $9.5 \leq E[K] \leq 11$ GB, the M plan with 15-GB option is recommended. If $E[K] \geq 11$ GB, the M_u plan is recommended.

For $I = 5$, the fixed payment of the M_u plan is \$330. The results of the primary price study in Figure 16 (a) show that if $K \leq 11.2$ GB, the M plan with 10-GB option is recommended. If $11.2 \leq K \leq 17$ GB, the M plan with 15-GB option is recommended. If $K \geq 17$ GB, the M_u plan is a better choice. Figure 16 (b) shows the expected payments $C_{P,5,J}$ against $E[K]$ for the M_u plan and the M plan with various options. In this figure, if $E[K] \leq 9$ GB, the M plan with 10-GB option is a better choice. If $9 \leq E[K] \leq 12.5$ GB, the M plan with 15-GB option is recommended. If $E[K] \geq 12.5$ GB, the M_u plan should be selected.

Similar to Section 5, the results we observed based on the measured data of CHT are more accurate than those of the primary price study. The primary price study optimistically favors the S_u and M_u plans, and the M plan with J - GB options with smaller J values.



(a) (b)

Figure 16: Comparing the Mu plan and the M plan with options 10-GB and 15-GB for $I = 5$. (a) The primary price study; (b) The expected payments ($C_{P,5,J}$)

Table 6: The *Mobile Share Value* Plan of AT&T Communications Inc. (Nov. 2014)

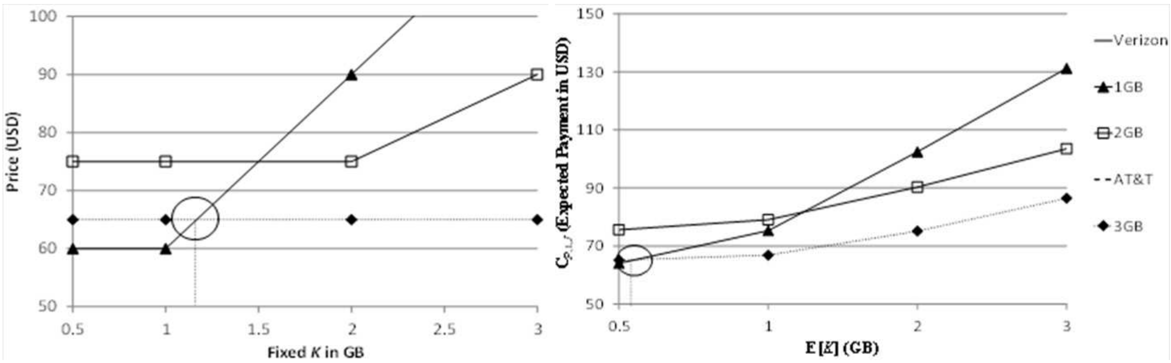
Monthly Data Allowance	300MB	1GB	2GB	4GB	6GB	10GB	30GB	40GB	60GB	80GB	100 GB
Monthly Account Access	$20	$25	$40	$70	$80	$100	$130	$150	$225	$300	$375
Data Overage	$20/ 300MB	$20/ 500MB	$15/1GB								

Devices	Smartphones	Basic Phones		Tablets& Connect Device	Internet Devices
Monthly Line Access	$40	$20 (300MB-6GB)	$15 (10GB-50GB)	$10	$20

7 Comparing the Verizon's and AT&T's Billing Plans

This section compares the non-shared and the shared data plans of AT&T with Verizon's. AT&T's non-shared plan (the November 2014 version) is called the *Individual* Plan (we abbreviate it as the S_a plan). The S_a plan has only one data allowance option of 3 GB, and the basic price is $65. If the amount of data consumed by the user is more than 3 GB, there will be an extra charge of $15 per 1GB. AT&T's shared data plan is called the *Mobile Share Value* Plan (the M_a plan) [2], where a group of users with up to ten devices may share an account. As listed in Table 6, the M_a plan offers 11 data allowance options from 300MB to 100GB. Like Verizon's M plan, the number of mobile devices also affects the price. For example, if the group of users applies to the 10-GB option with five devices (three smartphones, a tablet, and an Internet device), then they should pay $250 ($100 + $40×3+ $10 + $20) per month. Both the S_a and the M_a plans provide unlimited voice calls and SMS.

This section compares the billing plans of Verizon and AT&T using the analytic model with the measured data of CHT.



(a) (b)

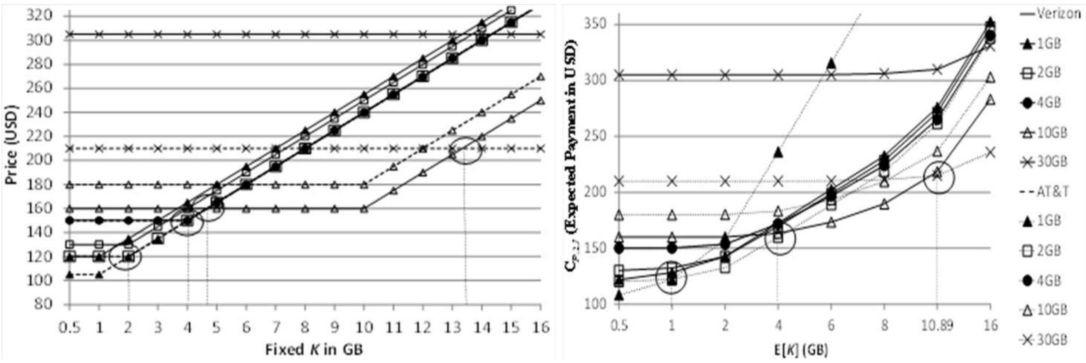
Figure 17: Comparing the S and the S_a plans for $I = 1$. (a) The primary price study; (b) The expected payments ($C_{P,1,J}$)

7.1 Comparison of the S and the S_a Plans

For $I = 1$, $K = K_1$, and Figure 17 (a) compares the S plan (the solid curves) with the S_a plan (the dashed curve) through the primary price study. In this figure, if $K \leq 1.2$ GB, the S plan with 1-GB option is recommended. If $K \geq 1.2$ GB, the S_a plan with 3-GB option is a better choice. Based on the measured data of CHT, Figure 17 (b) plots the curves for $C_{S,1,1}$ against E[K] (the S plan with 1-GB option), $C_{S,1,2}$ (the S plan with 2-GB option), and $C_{S_a,1,3}$ (the S_a plan with 3-GB option). The figure shows that if E[K] ≤ 0.55 GB, the S plan with 1-GB option is a better choice. If E[K] ≥ 0.55 GB, the S_a plan with 3-GB option is better than the S plans. Figure 17 (b) provides more accurate payment comparison than Figure 17 (a).

7.2 Comparison of the M and the M_a Plans

Figure 18 (a) conducts the primary price study to compare the M plan (the solid curves) and the M_a plan (the dashed curves) with J-GB options for $I = 24$, where $J = 1$, 2, 4, 10 and 30 GB, and $K_1 = K_2 = 1/2K$. For $J = 1$, 2, and 30 GB, the M_a plan with J-GB options is better than the M plan with the same J-GB options. For $J = 4$ GB, both the M_a and the M plans are the same. For $J = 10$ GB, the M plan is better than the M_a plan. For $K \leq 2$ GB, the M_a Plan with 1-GB option is a better choice. If $2 \leq K \leq 4$ GB, both the Ma plan with 1-GB and 2-GB options are recommended. For $4 \leq K \leq 4.6$ GB, the M plan with 4-GB option and the M_a plan with 1-GB, 2-GB, and 4-GB options are recommended. If $4.6 \leq K \leq 13.5$ GB, the M plan with 10-GB option is a better choice. For $K \geq 13.5$ GB, the M_a plan with 30-GB option should be selected. Figure 18 (b) shows the expected payments $C_{P,2,J}$ against E[K] for the M_a and the M plans with various options, where E[K] = E[$K_1 + K_2$]. The figure indicates that if E[K] ≤ 1 GB, the M_a plan with 1-GB should be selected. If $1 \leq E[K] \leq 4$ GB, the M_a plan with 2-GB option is a better choice. If $4 \leq E[K] \leq 10.89$ GB, the M plan with 10-GB option is recommended. If E[K] ≥ 10.89 GB, the M_a plan with 30-GB option is a better choice. For $I \geq 2$, the price curves in the primary price study and the $C_{P,I,J}$ curves are similar to those in Figure 18, and the details are omitted. Our study indicates that for the users with large or small data consumptions, the AT&T plans are better choices. For the users with medium data consumptions, the Verizon plans should be selected. Both the primary price study and the study with CHT measured data show the consistent results.



(a) (b)

Figure 18: Comparing the M and the M_a plans for $I = 2$. (a) The primary price study; (b) The expected payments ($C_{P,2,J}$)

8 Conclusions

In the recent years, the mobile telecom operators have offered shared, non-shared, limited, and unlimited billing plans for data services. To subscribe mobile services, the users need guidelines to select appropriate billing plans. The mobile operators typically provide rough billing plan suggestions for a group of users with the primary price study [10, 18]. However, the suggestions may be misleading in some cases. This paper proposed an analytical approach to enhance the billing plan suggestion. By considering the measured data usage of 900 users from a commercial mobile network in one year, our study indicates that the user behaviors must be taken into consideration to provide accurate billing plan suggestions.

The advantage of the primary price study is that the relationship between the price and the (fixed) consumed data K can be easily and quickly calculated. From the experiments conducted in this paper, the primary price study catches the trends of the relationship between the price and K. However, this approach does not capture the actual K behavior, and therefore cannot provide accurate results. By utilizing the actual data usage measured from a commercial network, we derived statistical distributions based on the measured data K, and proposed an analytic model to compute the actual payment $C_{P,I,J}$ for various billing plans P with the option J for a group of I users. The $C_{P,I,J}$ values derived from our model show consistent trends as the primary price study but with more accurate results, and therefore provide better suggestions for billing plan selection. Based on the primary price study and the proposed analytic model, we investigate the tradeoffs of various billing plans:

- Comparing shared and non-shared billing plans: For $I = 1$, the *Single Line* Plan is a better choice if $\mathrm{E}[K] \leq 5.445$ GB. For $I = 2$, the *Single Line* Plan is recommended if $\mathrm{E}[K] \leq 1.3$ GB. If $I \geq 3$, the *MORE Everything* Plan is a better choice. People may think that the *Single Line* Plan is the right choice for a single user. In fact, the *MORE Everything* Plan with an appropriate option is a better choice for a single user with a large data consumption.

- Comparing limited and unlimited billing plans: It is obvious that the *Unlimited Single Line* Plan and the *Unlimited MORE Everything* Plans are right choices for large data consumptions. On the other hand, limited billing plans should be selected for small and medium data usage. For $I = 1$, the *Limited Single Line* Plan is recommended if $\mathrm{E}[K] < 4.1$ GB. For multiple users, the *Limited MORE Everything* Plan is a better choice if $\mathrm{E}[K] < 7$ GB (for $I = 2$), $\mathrm{E}[K] < 9$ GB (for $I = 3$), $\mathrm{E}[K] < 11$ GB (for $I = 4$), and $\mathrm{E}[K] < 12.5$ GB (for $I = 5$).

- Comparing AT&T's and Verizon's billing plans: For the users with large or small data consumptions, the AT&T *Mobile Share Value* Plan is better choice. For the users with medium data consumptions, the Verizon *MORE Everything* Plan should be selected. Specifically, for $\mathrm{E}[K] \leq 4$ GB or $\mathrm{E}[K] \geq 10.89$ GB.

The above conclusions are drawn based on the measured data of CHT. We emphasize that our model can be easily extended for other mobile operators. To accommodate their billing plans, the mobile operator just derives the statistical distribution on the data usage measured from their own commercial network, and then applies this distribution to our models with the desired billing plans to provide appropriate suggestions for their subscribers.

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Appendices

A The Simulation Model

This section describes the simulation model for the M and the S plans. The simulation programs for M_u, S_u, M_a, and S_a are similar and omitted. Suppose that there are I users in a group. We compute the expected payments of the M and S plans for the m-th month, where $1 \le m \le 12$. This simulation experiments are used to validate against the analytic model in Section 4 to ensure the correctness of the analytic model. As described in Section 3, let K_i be the data usage consumed by user i in one month, which is drawn from a random number generator G_m. For $1 \le i \le I$, assume that K_i are independent and identically distributed (i.i.d.) random variables for the m-th month, where $1 \le m \le 12$. In our experiments, G_m is a Gamma random number generator with the mean and the variance derived from CHT measured data.

The simulation experiments are conducted for the M plan (Figure 19) and the S plan (Figure 20). The simulation replications of the M and the S plans repeat 106 times to generate stable results. Figure 19 illustrates the simulation flow chart of the M plan for the m-the month with the following steps:

Step 1. For $1 \le i \le I$, generate random numbers K_i from G_m (the data usage consumed by user i in one month). Set $Iteration$ to 0, and $C_{M,I,J,m}$ to 0.

Step 2. If $Iteration < 10^6$, the simulation is not complete and the execution flow proceeds to **Step 3**. Otherwise, **Step 8** is executed to terminate the simulation.

Step 3. Set K to the sum of K_i, where $1 \le i \le I$.

Step 4. If $K > J$, there is an additional payment for the extra data consumption K–J. Otherwise, the payment is the basic price for the monthly data allowance of J GB.

Step 5. $L_u(K)$ is computed as $\lceil \frac{K}{u} \rceil \times u$, and $N_m = \frac{\max[L_u(K)-J,0]}{u}$. The payment $C_{M,I,J,m}$ is incremented by $c_{M,I,J} + N_m \times o_{M,I,J}$.

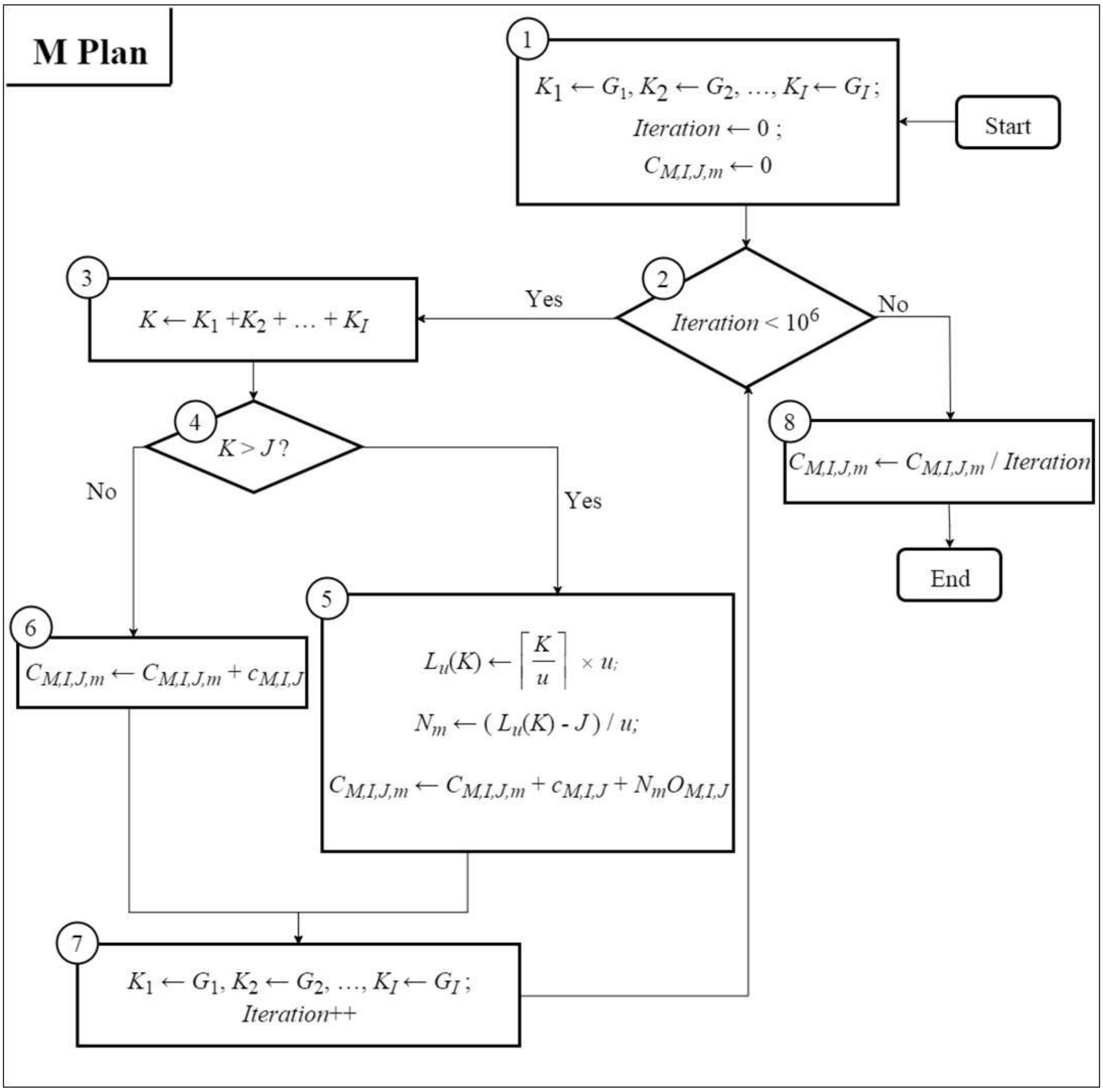

Figure 19: The simulation flow chart for the M plan of the m-th month for the I-user group with a monthly data allowance of J GB

Step 6. Increment the payment $C_{M,I,J,m}$ by $c_{M,I,J}$.

Step 7. This replication is completed. For $1 \le i \le I$, generate new random numbers K_i from G_m. Increment $Iteration$ by 1 and proceed to **Step 2**.

Step 8. $C_{M,I,J,m}$ is computed as $\frac{C_{M,I,J,m}}{Iteration}$, and the simulation is terminated.

Figure 20 illustrates the simulation flow chart of the S plan for the m-the month with the following steps:

Step 1. For $1 \le i \le I$, generate random numbers K_i from G_m (the data usage consumed by user i in one month). Set $Iteration$ to 0, and $C_{S,I,J,m}$ to 0.

Step 2. If I$teration < 10^6$, the simulation proceeds to **Step 3**. Otherwise, **Step 10** is executed to terminate the simulation.

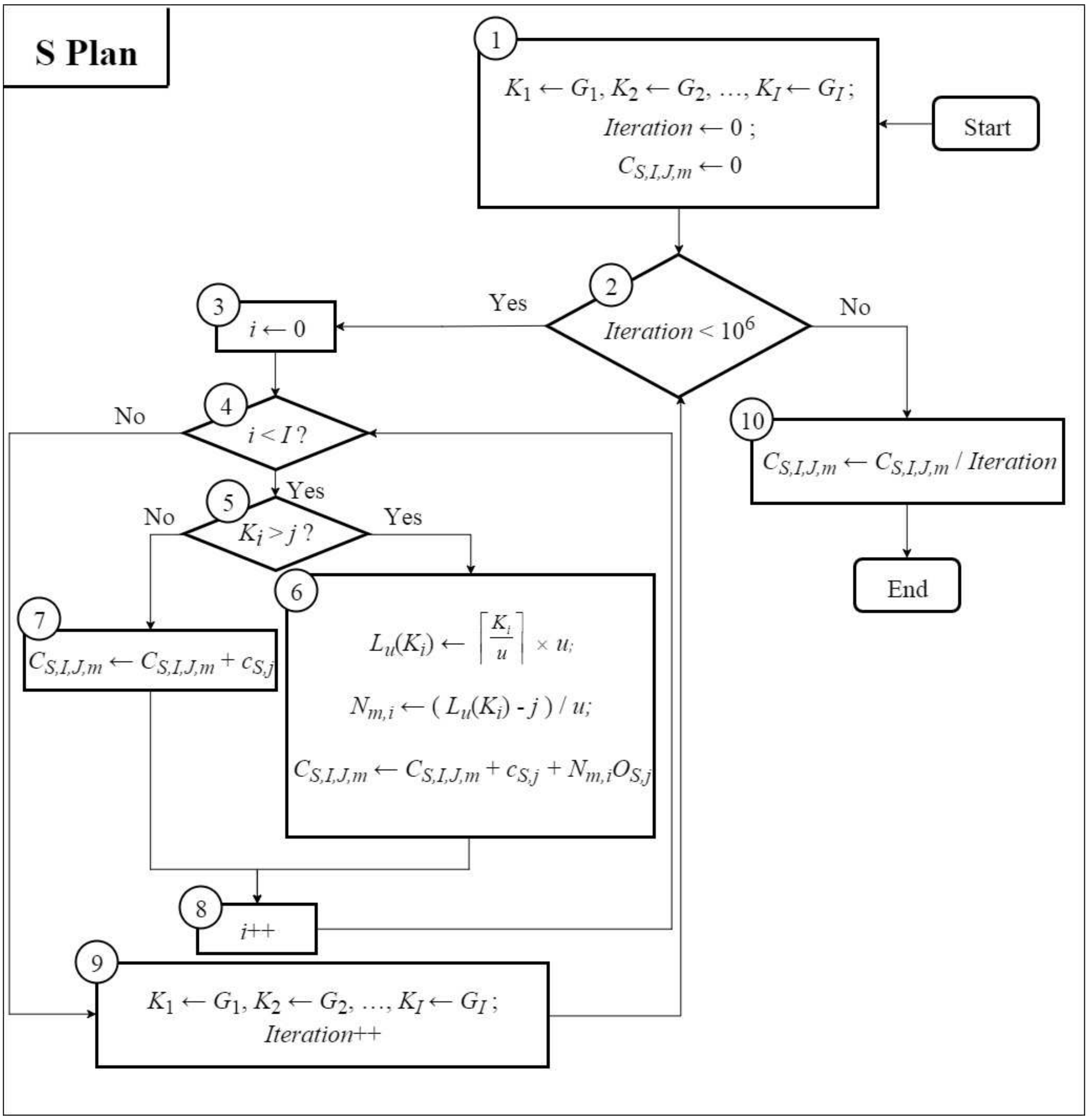

Figure 20: The simulation flow chart for the S plan of the *m*-th month where each of the *I* users applies for the S plan with a monthly data allowance of *j* GB

Step 3. Set *i* to 0. The counter *i* counts the number of users.

Step 4. If $i < I$, **Step 5** is executed to compute the payment of user *i*. Otherwise, **Step 9** is executed to compute the payment for the next replication.

Step 5. If $K_i > j$, there is an additional payment for the extra data consumption K_i–j. Otherwise, the payment is the basic price for the monthly data allowance of *j* GB.

Step 6. $L_u(K_i)$ is computed as $\lceil \frac{K_i}{u} \rceil \times u$, and $N_{m,i} = \frac{\max[L_u(K_i) - j, 0]}{u}$. The payment $C_{S,I,J,m}$ is incremented by $c_{S,j} + N_{m,i} \times o_{S,j}$.

Step 7. Increment the payment $C_{S,I,J,m}$ by $c_{S,j}$.

Step 8. Increment i by 1. **Step 4** is executed to compute the payment of the next user.

Step 9. This replication is completed. For $1 \leq i \leq I$, generate new random numbers K_i from G_m. Increment Iteration by 1 and proceed to **Step 2**.

Step 10. $C_{S,I,J,m}$ is computed as $\frac{C_{S,I,J,m}}{Iteratrion}$, and the simulation is terminated.